Subsidies in an Endogenous Growth Model with Capital-Embodied Technical Change

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Abstract

As robotics and artificial intelligence advance rapidly, automation-related policies have gained prominence in industrialized economies. Based on a growth model with capital-embodied technical change, this study investigates the effects of three distinct subsidy instruments: subsidies for intermediate goods, subsidies for automation, and subsidies for vertical innovation. Theoretical analysis reveals that while intermediate-good and automation subsidies promote long-run growth, innovation subsidies hinder it by crowding out automation. Calibrated to US data, our quantitative analysis shows that although innovation subsidies are detrimental to long-run growth, they generate a positive effect in the short run. We further compare the steady-state and transitional welfare effects, finding that the policy implications for automation and innovation subsidies shift when transition dynamics are considered, whereas the implications for intermediate-good subsidies remain unchanged. Finally, we characterize the dynamically optimal policy mix required to replicate the first-best outcome, demonstrating that the welfare loss from implementing the constrained long-run optimal policy is non-negligible. These results highlight the critical importance of transition dynamics in the design of optimal policy.

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Keywords: Automation; capital-embodied technical change; economic growth; subsidies

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1 Introduction

Modern technological progress is often embodied in improvements in capital equipment such as computers and machinery. As documented by Greenwood *et al.* (1997), embodied technical change has even accelerated after 1974, and capital-embodied technological change accounts for the majority of growth. In recent years, the arrival of robotic technology and generative artificial intelligence (AI) has intensified this pattern, reinforcing the importance of capital-embodied technical change in ongoing economic transformation. Moreover, these newly emerging technologies are widely regarded as the competitive frontier of the Fourth Industrial Revolution, and many countries have launched various programs and subsidies to develop AI and robotics technologies. For example, China's "14th Five-Year Plan" for robotics places great emphasis on innovation in robotics technology and industrial development. The updated "Key Special Program on Intelligent Robots" (2024) dedicates a budget of about 45.2 million US dollars specifically to promote fundamental frontier technologies (such as generative AI). Other economies such as the United States, Germany, and Korea have also launched public programs to support intelligent robotics and related technologies.[1]

These policies have contributed to recent breathtaking developments in robotic technology and AI, bringing about intensive automation. Yet such automation remains contentious: some contend that it will deliver unprecedented productivity gains and alleviate labor shortages caused by population decline, while others are pessimistic about the widespread job displacement and inequality it may incur. These concerns raise a set of questions: what are the economic effects of automation-promoting policies? More fundamentally, how should governments design subsidy policies when technical change is capital-embodied?

We explore the above questions based on the endogenous growth model put forward by Jones and Liu (2024), which features two dimensions of capital-embodied technological progress. The first one is referred to as horizontal innovation, which involves automated productions that were previously performed by labor. The second one is referred to as vertical innovation, which follows successful automation, emerges aimed at improving productivity to reduce the amount of capital required per unit of output. Consequently, we apply this framework to examine implications of three types of subsidy instruments that are commonly considered in existing literature: subsidies to the purchase of intermediate goods, subsidies to R&D (vertical innovation), and subsidies to automation (horizontal innovation).[2]

In this growth-theoretic framework, we obtain the following analytical results. First, a higher intermediate good subsidy has a positive effect on economic growth and the labor share of income. Intuitively, subsidizing the purchase of intermediate goods raises the demand for intermediate goods and thereby enhances the profit of automated firms. This motivates entrepreneurs

[1]The US National Science Foundation (NSF) supports fundamental research in intelligent robotics, focusing on applications across workplaces, healthcare, and domestic settings, with a requested 2024 budget of approximately 70 million US dollars. Germany's "High-Tech Strategy 2025 (HTS)" program and Korea's "4th Basic Plan on Intelligent Robots", committing an investment of 369.2 million and 128 million US dollars, respectively. (Data source: World Robotics R&D Programs 2025 report.)

[2]Throughout this study, we use R&D and vertical innovation interchangeably, and automation and horizontal innovation interchangeably, to refer to the same policy targets.

to invest in automation, which in turn promotes growth. At the same time, it raises the relative price of labor-intensive to capital-intensive intermediate goods by increasing the wage rate. As a result, the labor share of income rises given the gross complementarity between capital-intensive and labor-intensive goods. Second, a higher automation subsidy boosts economic growth and dampens the labor share of income. This is because automation subsidies reallocate resources towards automation, which accelerates economic growth. However, the expansion of automation allows more workers to be replaced by capital in production, which tends to reduce the labor share of income. Third, a higher R&D subsidy reduces economic growth and increases the labor share of income; it incentivizes more investment in vertical innovation, which crowds out resources allocated to automation and thereby hinders economic growth. Meanwhile, the vertical innovation advances the productivity of capital and lowers the relative price of capital-intensive goods, raising the labor share of income as a consequence.

Applying the US aggregate data, our quantitative analysis reveals several novel findings. First, subsidization in automation and R&D subsidies entail a trade-off between growth and welfare: a higher automation subsidy raises economic growth but lowers steady-state welfare by reducing the initial level of consumption, whereas a higher R&D subsidy raises steady-state welfare but stifles economic growth by crowding out automation. Therefore, these two instruments can be used complementarily in the long run as follows: automation subsidy stimulates economic growth, whereas R&D subsidy mitigates the adverse effect of increased automation on steady-state welfare, delivering moderate gains in both balanced growth and steady-state welfare.

Following the discussion of long-run policy implications, we also quantitatively examine the transitional dynamic effects of these subsidies. We find that their transitional dynamic effects on economic growth and the labor share of income are broadly consistent with the long-run counterparts, with one important exception on the short-run growth effect of R&D subsidy. In particular, although a higher R&D subsidy stifles long-run growth, it has a positive effect on growth in the short run. When these dynamic effects are taken into account, welfare policy implications differ from those based on steady-state analysis, except in the case for the intermediate-good subsidy, which generates no transition dynamics. For automation subsidy, the short-run welfare gains may outweigh its long-run welfare loss, leading to a reversal in policy implications. Specifically, the transitional welfare analysis suggests a welfare-maximizing automation subsidy of approximately 51%, whereas the steady-state analysis argues against any such subsidy. For R&D subsidy, the intertemporal trade-off between short-run welfare losses and long-run gains results in an inverted-U shaped transitional welfare effect. This implies a moderate welfare-maximizing R&D subsidy of 33%, which is lower than the rate suggested by the steady-state analysis.

Finally, we explore the optimal mix of three policy instruments needed to achieve the social optimum by neutralizing monopoly pricing distortions and internalizing the externalities of automation and R&D. Our results show that the optimal intermediate-good subsidy is time-invariant. By contrast, the optimal automation subsidy and the optimal R&D subsidy are both time-varying and adjust monotonically towards their steady-state levels. Specifically, if the economy begins at the calibrated moments, these subsidies start above their long-run levels and

gradually decline over time, correcting the underinvestment in both types of innovations. Given the practical challenges of tracking an economy's exact position along a transition path, we evaluate an alternative regime where policymakers are constrained to time-invariant, long-run optimal rates. Our findings reveal that such a simplification entails a non-negligible welfare loss, suggesting that an effective policy framework cannot overlook transition dynamics in favor of long-run steady-state targets.

2 Literature review

This study is closely related to the vast literature that justifies the rationale for government intervention in innovation activities in canonical innovation-driven growth models. Seminal studies, such as Segerstrom (2000), Şener (2008), Impullitti (2010), Chu *et al.* (2016), and Chu and Wang (2022), have extensively analyzed the effects of R&D subsidies in models where innovation unfolds through variety expansion, quality improvement, or both. Recently, the thriving of robotic technology and AI have brought increasing attention to another form of innovation: automation, which enables capital to replace labor. Chu *et al.* (2023) introduces automation into a Schumpeterian growth model to explore the effects of R&D and automation subsidies. In their innovation-automation cycle, economic growth depends on innovation in the form of quality improvement, while the share of automated industries endogenously determines the capital intensity.[3] They find that R&D subsidies increase innovation and growth but decrease capital intensity, whereas automation subsidies have the opposite effects. By contrast, in our framework, automation serves as the primary engine of growth, while quality-improving innovation mainly functions to balance capital intensity. As a result, we obtain different policy implications: automation subsidies stimulate growth, whereas R&D subsidies stifle it by crowding out automation. In addition, rather than focusing solely on the long-run impacts, we also examine the transitional dynamic effects of these policies and characterize the first-best optimal policy mix, thereby providing a more comprehensive understanding of policy implications.

Moreover, the studies mentioned above invariably feature purely labor-augmenting technical change. However, this formulation overlooks a crucial reality: major technological advances are often intrinsically embodied in physical capital, and evidence suggests that capital-embodied technological change accounts for the majority of growth (see, for example, Greenwood *et al.* 1997 and Sakellaris and Wilson 2004). Building on the model of Jones and Liu (2024), which formalizes two types of capital-embodied technical advance, we extend this framework by incorporating three policy instruments: subsidies to the purchase of intermediate goods, subsidies to automation, and subsidies to vertical innovation, to revisit the growth and welfare implications of subsidies on innovation activities under capital-embodied technical change. Our study also complements Jones and Liu (2024) by explicitly analyzing the transitional dynamics of this model, which allows us to explore the macroeconomic effects of government policies over time

[3]Innovation in the form of a quality improvement can arrive at an automated or unautomated industry. When an innovation arrives at an automated industry, the industry becomes unautomated until the next successful automation arrives.

and the dynamically optimal design of policy combination.

This study also relates to the growing literature on automation and innovation.[4] A pioneering study by Zeira (1998) models automation as capital-labor substitution, laying the foundation for subsequent research on automation. However, the implication of the rising growth rate and capital share with automation contradicts the stylized facts documented by Kaldor (1961).[5] Building on this fact, Acemoglu and Restrepo (2018) and Hémous and Olsen (2022) introduce directed technological change involving automation and the creation of new varieties to allow for balanced growth, investigating the effects of automation on the labor market and income inequality in the process of economic growth.[6] Subsequently, Acemoglu *et al.* (2020) propose that the US tax system is biased against labor and in favor of capital, showing that the constrained optimal policy levies an automation tax to discourage the automation of marginal tasks that bring little productivity benefits but significant displacement of labor. Chu *et al.* (2023) propose that whether the government should subsidize automation depends on how welfare gains and losses are evaluated across different agents in the economy. Our paper complements this line of interesting research by providing an additional perspective that considers transitional dynamics. We find that subsidizing automation alone should be avoided from the steady-state welfare perspective. However, a moderate automation subsidy rate is necessary when transitional dynamics are taken into account. Moreover, our first-best policy analysis shows that the desirability and magnitude of automation subsidies depend on the current state of the economy.

3 The Model

We follow Jones and Liu (2024) to extend the quality-ladder endogenous growth model with capital-embodied technological change by allowing capital inputs to replace labor with a process of automation. Specifically, the model features two types of innovations: vertical innovation (R&D) improving productivity in automated industries, whereas horizontal innovation automating the production of industries. Moreover, we incorporate into this model with subsidies to the purchase of intermediate goods and expenditures on R&D and automation. This study analyzes the growth and welfare implications of subsidization by controlling one or a mix of these policy instruments.

3.1 Households

Suppose that the economy admits a unit continuum of identical households, and the lifetime utility function of a representative household is given by

$$U_0 = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{1}$$

[4]See Aghion *et al.* (2017) for a comprehensive discussion of this literature.

[5]The Kaldor's fact shows that the growth rate and capital shares are relatively stable over time.

[6]Prettner and Strulik (2020) analyze automation and inequality by conceptualizing R&D as the engine for producing automation patents and introducing endogenous education.

where c_t represents consumption of the final good at time t, and $\rho > 0$ is the discount rate. There is no population growth in the economy, and the population size is assumed to be L. A household inelastically supplies one unit of labor to earn a wage at the rate of w_t. Each household chooses consumption c_t to maximize (1) subject to the following asset-accumulation equation:

$$\dot{a}_t = r_t a_t + w_t - c_t + T_t, \tag{2}$$

where a_t is the real asset value, r_t is the real interest rate, and T_t is the lump-sum transfer. Performing standard dynamic optimization yields the familiar Euler equation such that

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

3.2 Final good

Competitive firms produce the final good Y_t by combining a unit interval of intermediate goods according to the following function with a constant elasticity of substitution (CES):

$$Y_t = v \left[\int_0^1 y_t(i)^\sigma di \right]^{1/\sigma}, \quad \sigma < 0, \tag{4}$$

where v represents factor productivity and $y_t(i)$ denotes intermediate good indexed by $i \in [0, 1]$.[7] The assumption $\sigma < 0$ implies that the elasticity of substitution is less than unity; namely, intermediate goods are gross complements.

Then the profit of the competitive final-good producers is given by

$$\hat{\pi}_t = v \left[\int_0^1 y_t(i)^\sigma di \right]^{1/\sigma} - (1 - s^x) \int_0^1 p_t(i) y_t(i) di, \tag{5}$$

where $p_t(i)$ is the price of $y_t(i)$, and $s^x \in (0, 1)$ is the subsidy rate to the purchase of intermediate goods. Using equation (5) for profit maximization yields the conditional demand function of each intermediate good $y_t(i)$ such that

$$y_t(i) = v^{\frac{\sigma}{1-\sigma}} [(1 - s^x) p_t(i)]^{\frac{1}{\sigma - 1}} Y_t. \tag{6}$$

3.3 Intermediate goods

There is a unit continuum of industries indexed by $i \in [0, 1]$, producing differentiated intermediate goods. Each intermediate good can be produced with either labor or automation. Suppose that industries in interval $i \in [0, \beta_t]$ are automated, allowing them to utilize capital in their production processes. Following Jones and Liu (2024), the production possibilities for the

[7] We adopt the interpretation from Zeira (1998), viewing $y_t(i)$ as intermediate goods, while Acemoglu and Restrepo (2018) refer to these as tasks.

intermediate goods are given by

$$y_t(i) = \begin{cases} Al_t(i) & \text{for all } i \in [0,1] \\ z_t(i)^{\frac{\sigma-1}{\sigma}} x_t(i) & \text{for all } i \in [0,\beta_t] \end{cases} \tag{7}$$

where $l_t(i)$ is a labor input, and A is the labor productivity parameter.[8] $x_t(i)$ is a capital input that can be made at a marginal cost of ψ units of the final good. Following the standard approach in the quality-ladder model, we assume that capital inputs are fully depreciated, and we normalize the cost of capital such that $\psi = v$. Furthermore, the productivity of $x_t(i)$ is determined by its own quality $z_t(i)$. Given the productivity level $z_t(i)$, the marginal cost of the leader in an automated industry $i \in [0,\beta_t]$ is $mc_t(i) = \psi / z_t(i)^{\frac{\sigma-1}{\sigma}}$. For automated industries $i \in [0,\beta_t]$, capital or labor input can be used in their production processes. However, we are interested in the equilibrium where automated firms (i.e. $i \in [0,\beta_t]$) do indeed use capital inputs. To achieve this, a technical condition is imposed to ensure that capital-based production is less costly than labor-based production in these automated industries:

$$w_t \geq \psi A z_t(i)^{\frac{1-\sigma}{\sigma}}. \tag{8}$$

3.3.1 Non-automated industries

Given the labor-based production technology in (7), non-automated intermediate goods are produced in competitive markets, and the price for any non-automated intermediate good is

$$p_t(i) = \frac{w_t}{A}, \quad i \in (\beta_t, 1] \tag{9}$$

Given this price, the demand for these non-automated intermediate goods is identical. Automated industries (i.e., $i \in [0,\beta_t]$) indeed use capital inputs, while labor inputs are exclusively used in non-automated industries (i.e., $i \in (\beta_t, 1]$). Therefore, the equilibrium labor allocation is given by

$$l_t(i) = \frac{L}{1-\beta_t}, \quad i \in (\beta_t, 1], \tag{10}$$

where L denotes the total labor supply. The output for any labor-performed intermediate good is

$$y_t(i) = \frac{AL}{1-\beta_t}, \quad i \in (\beta_t, 1]. \tag{11}$$

Then combining (6), (9) and (11), we can obtain the equilibrium wage rate such that

$$w_t = \frac{(Av)^\sigma}{1-s^x} \left[\frac{L}{(1-\beta_t)Y_t} \right]^{\sigma-1}. \tag{12}$$

[8] To emphasize that technological progresses in the model is purely capital-embodied, the labor productivity is held constant.

3.3.2 Automated industries

For industries $i \in [0, \beta_t]$, producers of automated goods possess market power. The profit-maximizing price involves a constant markup over the marginal cost $\psi / z_t(i)^{\frac{\sigma-1}{\sigma}}$ such that

$$p_t(i) = \mu \frac{\psi}{z_t(i)^{\frac{\sigma-1}{\sigma}}}, \quad i \in [0, \beta_t], \tag{13}$$

where $\mu > 1$ is a policy instrument that captures the degree of patent breadth set by patent authority. Given this price, using (6), the demand for the automated good $y_t(i)$ is

$$y_t(i) = v^{-1}[(1 - s^x)\mu]^{\frac{1}{\sigma-1}} z_t(i)^{-\frac{1}{\sigma}} Y_t, \quad i \in [0, \beta_t]. \tag{14}$$

Equating the demand and supply of this good yields:

$$x_t(i) = v^{-1}[(1 - s^x)\mu]^{\frac{1}{\sigma-1}} \frac{Y_t}{z_t(i)}. \tag{15}$$

Using (13), (14) and (15), the monopolistic profits in industry i is given by

$$\pi_t^v(i) = (1 - s^x)^{\frac{1}{\sigma-1}} \Theta \frac{Y_t}{z_t(i)}, \tag{16}$$

where $\Theta \equiv \mu^{\frac{\sigma}{\sigma-1}} - \mu^{\frac{1}{\sigma-1}}$ captures the markup effects.

Finally, the capital-productivity terms are aggregated using the index

$$Z_t = \left[\frac{1}{\beta_t} \int_0^{\beta_t} \frac{1}{z_t(i)} di \right]^{-1}, \tag{17}$$

which is the harmonic average of the capital-productivity technology terms.

3.4 Innovations

This model characterizes two types of innovations related to capital-embodied technology: horizontal innovation and vertical innovation. On the one hand, horizontal innovation involves firms automating industries that were formerly performed by labor. On the other hand, vertical innovation involves firms that improve capital productivity $z_t(i)$ of each automated process through a quality ladder. In other words, vertical innovation can only emerge in automated industries.

3.4.1 Vertical innovation

In the vertical innovation process, innovators attempt to increase the productivity of a given automated industry by a proportional amount ϕ. Following standard vertical growth models (e.g., Aghion and Howitt 1992), a successful innovator sells its invention at a price that corre-

sponds to the expected present value of the monopoly rents generated by the patent. For a given productivity $z_t(i)$, the flow value of profits from such an innovation is presented by (16). However, the firm will lose these profits through creative destruction when another firm develops a new generation good along the same line, which occurs at an arrival rate of $q_t^v(i)$. Defining by $V_t(i)$ the real value of a firm who holds the latest innovation in line i, we have the following Hamilton–Jacobi–Bellman (HJB) equation:

$$r_t V_t(i) = \pi_t^v(i) + \dot{V}_t(i) - q_t^v(i) V_t(i). \tag{18}$$

This equation implies that the return on $V_t(i)$ equals the sum of monopolistic profit $\pi_t^v(i)$, capital gain $\dot{V}_t(i)$ and the potential loss $q_t^v(i)V_t(i)$ due to creative destruction.

Competitive entrepreneurs employ $d_t^v(i)$ units of final good to engage in innovation across all automated industries i (where $i \in [0, \beta_t]$). The arrival rate of innovation in industry i is given by

$$q_t^v(i) = \xi^v \left(\frac{z_t(i) d_t^v(i)}{Y_t} \right)^{\alpha}, \tag{19}$$

where ξ^v is an innovation productivity parameter that quantifies the ease of achieving innovation in the vertical direction. The parameter $\alpha \in (0,1)$ captures the overall effects from diminishing returns to R&D expenditure and positive intertemporal knowledge spillovers.

Free entry into this R&D sector implies that

$$q_t^v(i) V_t(i) = (1 - s^v) d_t^v(i), \tag{20}$$

where $s^v < 1$ is the R&D subsidy rate.[9] Combining (19) and (20) yields:

$$V_t(i) = \frac{(1-s^v) d_t^v(i)}{q_t^v(i)} = \frac{(1-s^v) q_t^v(i)^{\frac{1}{\alpha}-1} Y_t}{\xi^{v\frac{1}{\alpha}} z_t(i)}. \tag{21}$$

Following the standard literature, we focus on a symmetric equilibrium where the arrival rate of vertical innovation is identical across automated industries such that $q_t^v(i) = q_t^v$. Consequently, combining (17) and (19) yields the aggregate expenditure on vertical innovation, denoted by D_t^v, which is given by

$$D_t^v = \int_0^{\beta_t} d_t^v(i) di = \left(\frac{q_t^v}{\xi^v} \right)^{\frac{1}{\alpha}} \beta_t Z_t^{-1} Y_t. \tag{22}$$

3.4.2 Horizontal innovation

Horizontal innovation targets unautomated industries (i.e., $i \in (\beta_t, 1]$). There are also competitive entrepreneurs who invest $d_t^h(i)$ units of the final good to perform automation in those

[9] When $s^v < 0$, it acts as a tax rate on vertical innovation.

currently unautomated industries. The arrival rate for a new automation is given by

$$q_t^h(i) = \xi^h \left(\frac{z_t(i) d_t^h(i)}{Y_t} \right)^{\alpha}, \tag{23}$$

where ξ^h represents the efficiency of horizontal innovation. $\alpha \in (0,1)$ measures the degree of diminishing returns to horizontal innovation expenditures.

Furthermore, we assume that each new automation adoption is associated with an initial technology level, such that the productivity of the newly automated industries (i.e., $i = \beta_t$) follows the process:

$$z_t(\beta_t) = h(1 - \beta_t), \tag{24}$$

where h is constant.[10] This specification implies that the initial automation quality is falling as β_t rises. That is, automation addresses easier tasks first; industries that are automated later are inherently more complex, requiring more time to achieve successful automation and exhibiting lower initial productivity.

Upon the success of horizontal innovation, the newly automated industries enter a vertical phase, in which further vertical innovation enhances the productivity level $z_t(i)$ according to the process as described previously. In this case, we also assume that a markup μ defines the flow profits from the horizontal innovation as in (16). Consequently, the present value of this innovation is again given by (18). Therefore, the zero-profit entry condition is

$$q_t^h(i) V_t(i) = (1 - s^h) d_t^h(i), \tag{25}$$

where $s^h < 1$ denotes the subsidy rate to automation.[11]

Substituting (23) and (24) into (25), the present value of automation $V_t(i)$ can be written as:

$$V_t(i) = \frac{(1 - s^h) q_t^h(i)^{\frac{1}{\alpha} - 1} Y_t}{\xi^{h \frac{1}{\alpha}} h (1 - \beta_t)}. \tag{26}$$

Moreover, combing (23) and (24), the aggregate spending on horizontal innovation, denoted by D_t^h, is given by[12]

$$D_t^h = \int_{\beta_t}^1 d_t^h(i) di = \left(\frac{q_t^h}{\xi^h} \right)^{\frac{1}{\alpha}} h^{-1} Y_t. \tag{27}$$

[10] For simplicity and tractability, we assume that this process determines the initial technology level. Alternatively, one can consider settings where the initial productivity of the newly-automated industry is stochastic.

[11] When $s^h < 0$, it implies a tax rate on automation.

[12] Recall that horizontal innovation aims to industries that are not yet automated, i.e., those industries in the interval $i \in (\beta_t, 1)$. Upon success, the newly automated task is, for simplicity, re-indexed as $i = \beta_t$ and its initial productivity of these industries is given by (24). That is, all non-automated industries are treated as symmetric such that $q_t^h(i) = q_t^h$ for $i \in (\beta_t, 1)$.

3.5 Government

Suppose that the government can employ subsidies for the purchase of intermediate goods and innovations by choosing a set of subsidy rates $\{s^x, s^v, s^h\}$. The balanced-budget condition is given by

$$T_t = s^x \int_0^1 p_t(i) y_t(i) di + s^v \int_0^{\beta_t} d_t^v(i) di + s^h \int_{\beta_t}^1 d_t^h(i) di, \tag{28}$$

where the left-hand side represents tax revenues collected from households, while the right-hand side denotes the fiscal expenditures used to subsidize the purchase of intermediate goods, vertical innovation (R&D), and horizontal innovation (automation), respectively. Consequently, the government can utilize subsidy (or tax) instruments to influence resource allocation and guide the market economy toward a more efficient outcome.

3.6 Aggregation

Substituting (11) and (14) into (4) yields the total output such that

$$Y_t = \frac{vAL\left(1-\beta_t\right)^{\frac{1-\sigma}{\sigma}}}{\left[1-\left((1-s^x)\mu\right)^{\frac{\sigma}{\sigma-1}}\beta_t Z_t^{-1}\right]^{1/\sigma}}. \tag{29}$$

Using (15), the aggregate expenditure on final good used to produce capital is given by

$$I_t = \int_0^{\beta_t} \psi x_t(i) di = \left[\mu(1-s^x)\right]^{\frac{1}{\sigma-1}} \beta_t Z_t^{-1} Y_t, \tag{30}$$

which implies that the capital intensity is

$$\chi^I = \frac{I_t}{Y_t} = \left(\mu(1-s^x)\right)^{\frac{1}{\sigma-1}} \beta_t Z_t^{-1}. \tag{31}$$

Given that the profit function for the final-good firm is $Y_t = (1-s^x)(w_t L + I_t + \Pi_t)$, where Π_t is the aggregate profit across automated lines, the labor income share can be written as:

$$S_{L_t} = \frac{w_t L}{w_t L + I_t + \Pi_t} = (1-s^x)\frac{w_t L}{Y_t}. \tag{32}$$

Finally, substituting (12) and (29) into (32) yields the labor share of income such that

$$S_{L_t} = 1 - \left[(1-s^x)\mu\right]^{\frac{\sigma}{\sigma-1}} \beta_t Z_t^{-1}. \tag{33}$$

4 Solving the Model

This section proceeds to solve the model. Section 4.1 defines the decentralized equilibrium. Section 4.2 characterizes the balanced growth path (BGP) of the model and demonstrates the

balanced-growth equilibrium properties.

4.1 Decentralized equilibrium

The equilibrium consists of a time path of prices $\{w_t, r_t, p_t(i), V_t(i)\}_{t=0}^{\infty}$ and a time path of allocations $\{c_t, a_t, Y_t, y_t(i), x_t(i), l_t(i), d_t^h(i), d_t^v(i)\}_{t=0}^{\infty}$, which satisfy the following conditions at each instance of time:

- the representative household maximizes the utility taking prices $\{w_t, r_t\}$ as given;
- competitive final-good firms produce $\{Y_t\}$ to maximize profits taking $\{p_t(i)\}$ as given;
- competitive non-automated intermediate-good firm i (where $i \in (\beta_t, 1]$) produces $\{y_t(i)\}$ and chooses $\{l_t(i), p_t(i)\}$ to maximize profit taking $\{w_t\}$ as given;
- monopolistic automated intermediate-good firm i (where $i \in [0, \beta_t]$) produces $\{y_t(i)\}$ and chooses $\{x_t(i), p_t(i)\}$ to maximize profit;
- competitive innovation entrepreneurs choose $\{d_t^h(i), d_t^v(i)\}$ to maximize expected profit taking $\{V_t(i)\}$ as given;
- the labor market clears such that $L = \int_{\beta_t}^{1} l_t(i)di$;
- the capital market clears such that $I_t = \psi \int_0^{\beta_t} x_t(i)di$;
- the final-good market clears: $Y_t = Lc_t + I_t + D_t^h + D_t^v$;
- the value of inventions adds up to households' asset value such that $\int_0^{\beta_t} V_t(i)di = a_t L$;
- the government balances the fiscal budget.

4.2 Balanced growth path

As demonstrated in Jones and Liu (2024), two macro-level conditions are required for the economy to present a balanced growth path. First, the condition $\beta_t Z_t^{-1} = \kappa$ (defined as BGP1) must hold for a constant κ, suggesting a constant labor share of income (see equation (33)). Second, equation (29) implies that with a constant labor income share, income per capita grows at the rate of $g = \frac{1-\sigma}{\sigma} g_{1-\beta_t}$. The BGP thus requires that the fraction of non-automated industries is declining at a constant rate of $g_{1-\beta_t} = -q^h$ (defined as BGP2). Under these two BGP technology conditions, we can now examine the BGP equilibrium where g, r, q_t^v, and q_t^h are all constant.

Considering the vertical-innovation sector, the value of an innovation along the BGP can be written as

$$V(i) = \frac{\pi^v(i)}{r + q^v - g} = \frac{\Theta(1 - s^x)^{\frac{1}{\sigma-1}}}{r + q^v - g} \frac{Y}{z(i)}. \tag{34}$$

Substituting (34) and (19) into (20), the free-entry condition for R&D on the balanced growth path becomes

$$\frac{\xi^v \Theta (1-s^x)^{\frac{1}{\sigma-1}}}{r+q^v-g} = (1-s^v)\left(\frac{z(i)d^v(i)}{Y}\right)^{1-\alpha}, \quad i \in [0,\beta_t]. \tag{35}$$

This equation indicates that the ratio $z(i)d^v(i)/Y$ remains constant on the BGP. This suggests that more advanced industries, which have smaller shares of GDP, attract less effort in equilibrium. Then, using (35) and (19), the equilibrium arrival rate of R&D can be written as:

$$q^v = \frac{\Theta (1-s^x)^{\frac{1}{\sigma-1}}}{1-s^v} {\xi^v}^{\frac{1}{\alpha}} {q^v}^{\frac{\alpha-1}{\alpha}} - (r-g). \tag{36}$$

Similarly, for the horizontal-innovation sector, substituting (34) and (19) into (20) yields:

$$\frac{\xi^h \Theta (1-s^x)^{\frac{1}{\sigma-1}}}{r+q^v-g} = (1-s^h)\left(\frac{z(i)d^h(i)}{Y}\right)^{1-\alpha}, \quad i \in (\beta_t,1]. \tag{37}$$

As with the logic of vertical innovation, equation (37) implies a constant ratio of $z(i)d^h(i)/Y$ along the BGP, which leads to a constant arrival rate of automation among unautomated industries. Combining (37) with (23), we obtain

$$q^v = \frac{\Theta (1-s^x)^{\frac{1}{\sigma-1}}}{1-s^h} {\xi^h}^{\frac{1}{\alpha}} {q^h}^{\frac{\alpha-1}{\alpha}} - (r-g). \tag{38}$$

Consequently, equations (36) and (38) imply the following ratio of horizontal-vertical innovation rate:

$$\frac{q^h}{q^v} = \left(\frac{1-s^h}{1-s^v}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}. \tag{39}$$

We summarize the above results as follows.

Lemma 1. *On the balanced growth path (BGP), the hazard rates for R&D (q^v) and for horizontal innovation (q^h) are both constant. Their ratio is given by* $\frac{q^h}{q^v} = \left(\frac{1-s^h}{1-s^v}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}$.

Proof. Proven in the text. □

The arrival rate of automation relative to R&D (q^h/q^v) increases in the relative ease of innovation in these two directions (ξ^h/ξ^v) and the automation subsidy (s^h), while decreases in the R&D subsidy (s^v). With Lemma 1, we can revisit the conditions that determine the BGP and the technology adoption condition, where the newly automated technologies are more effective.

Recall that each automated industries increases its productivity proportionally by an amount ϕ with an arrival rate q^v on the BGP. Besides, the initial technology level of newly automated

industries is determined by (24). The first condition for BGP can be pinned down as[13]

$$\beta_t Z_t^{-1} = \frac{q^h}{h\phi q^v} = \frac{1}{h\phi}\left(\frac{1-s^h}{1-s^v}\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}. \tag{40}$$

The ratio of the technology indices $\beta_t Z_t^{-1}$ evolves in a constant ratio that increases with the automation subsidy rate but decreases with the R&D subsidy rate. This constant ratio further implies that balanced growth emerges if Z_t follows the same dynamics of as β_t. Specifically, along the BGP, the share of non-automated industries $1-\beta_t$ shrinks proportionally with time and approaches its limit (i.e., $\beta_t \to 1$). Similarly, Z_t shrinks to its limit $1/\kappa$ at a constant rate.[14]

In addition, from (31), this constant ratio of the technology indices $\beta_t Z_t^{-1}$ implies that the capital intensity is stable along the BGP. Specifically, two offsetting effects contribute to a constant capital intensity. On one hand, the fraction of automated industries that are performed by capital inputs β_t increases the capital intensity. On the other hand, the productivity index across these industries Z_t exerts a share-reducing effect. This negative effect arises because $\sigma < 0$: intermediate outputs are gross complements. As the productivity of a given sector advances, its price tends to fall relatively quickly, leading to a decline in the GDP share of each automated sector.

Equations (33) and (40) also indicate that the labor share remains constant along the BGP. With a constant labor share, steady state growth occurs when the fraction of unautomated industries is declining at a constant rate. From (29), the growth rate of income per capita is

$$g = \frac{\sigma-1}{\sigma} q^h. \tag{41}$$

Next, we examine the condition required for the adoption of all automated technologies (as opposed to using labor). Specifically, newly automated technologies must be sufficiently productive initially.[15] Then, the technology adoption condition, as assumed in (8) can be verified as follows.

Lemma 2. *To ensure that all automated technologies can be adopted on the BGP, we assume* $h \geq (1-s^x)^{\frac{\sigma}{\sigma-1}}\left[1+\mu^{\frac{\sigma}{\sigma-1}}\frac{1}{\phi}\left(\frac{1-s^h}{1-s^v}\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}\right]$.

Proof. See Appendix A.2. □

[13] See Appendix A.1 for the details.

[14] Note that the balanced growth path occurs when $1-\beta_t$ and $1/\kappa - Z_t$ decline at a constant rate $-q^h$, rather than when β_t and Z_t grow at a constant rate. Additionally, the model features "limited innovation, limitless growth", where a balanced growth can continue forever even though technology indices β_t and Z_t converge to finite limits. This occurs because each innovation step covers only a fraction of the remaining distance to the technological frontier. As a result, an infinite number of steps in the innovation process never fully close the finite gap to the technological limit. This dynamic parallels the logic of Zeno's Paradox, where one takes infinite steps yet never completes a journey of finite length.

[15] All automated technologies will be used so long as the newly automated technology is adopted.

5 Effects of Subsidies

In this section, we first analyze the growth effects of different subsidization instruments, including subsidies to the purchase of intermediate good and subsidies to automation and R&D. Subsequently, we explore the impacts of these three types of subsidies on the labor share of income and social welfare.

5.1 Economic growth

Combining (3) and (36) yields

$$q^v = \frac{\Theta(1-s^x)^{\frac{1}{\sigma-1}}}{1-s^v}\xi^{v\frac{1}{\alpha}}q^{v\frac{\alpha-1}{\alpha}} - \rho, \tag{42}$$

which determines the equilibrium arrival rate of vertical innovation q^{v^*}. Specifically, the left-hand side of this equation is a linear function of q^v; it starts at the origin, increases monotonically with q^v, and gradually approaches to infinity, as shown in the blue line in Figure 1. In contrast, the right-hand side is a decreasing function of q^v. It approaches infinity as $q^v \to 0$ and declines monotonically, eventually crossing zero for sufficiently large values of q^v, which corresponds to the orange line in Figure 1. Thus, these two functions intersect at a single positive value, denoted as q^{v^*}. In other words, there exist a unique positive solution for equation (42).

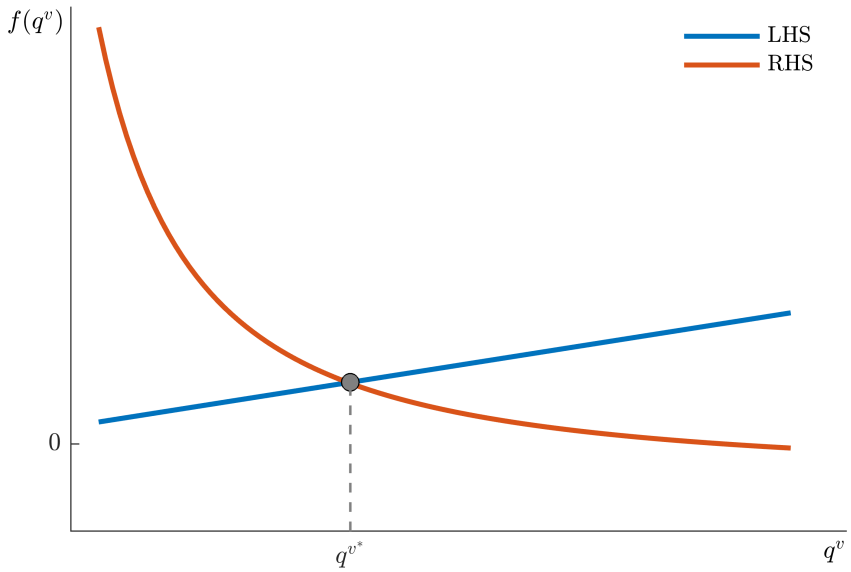


Fig. 1. Equilibrium for the arrival rate of innovation q^v

According to equation (42), subsidies to intermediate goods s^x and to vertical innovation s^v affect the arrival rate of vertical innovation q^v, while subsidies to horizontal innovation s^h has no

impact on q^v. The intermediate-good subsidy s^x shifts the orange line rightward, resulting in an increase in q^v. This occurs because an increase in s^x raises the demand for intermediate goods by effectively lowering the cost to final-good producers, enhancing the profitability of automated firms, which in turn incentivizes more investment in vertical innovation. Similarly, a higher R&D subsidy rate s^v raises q^{v^*} by moving the orange line to the right. Intuitively, an increases in R&D subsidy rate s^v decreases the cost of vertical innovation, raising the benefit of innovation. Hence, the economy tends to allocate more resources to this sector, which in turn leads to a higher arrival rate of innovation q^{v^*}, as shown in equation (19).

From Lemma 1, we can see that a unique positive value of q^{v^*} implies a unique positive arrival rate of automation q^{h^*}. Substituting (39) into (42) yields

$$q^h \left(\frac{1-s^h}{1-s^v}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{\alpha-1}} = \frac{\Theta(1-s^x)^{\frac{1}{\sigma-1}}}{1-s^h} \xi^{h\frac{1}{\alpha}} q^{h\frac{\alpha-1}{\alpha}} - \rho. \tag{43}$$

This equation determines the equilibrium arrival rate of automation q^{h^*}. As shown in Figure 2, the left-hand side is a linear increasing function in q^h whereas the right-hand side is a decreasing function. The intersection of these two functions yields the solution of the above equation, namely, the equilibrium arrival rate of automation q^{h^*}.

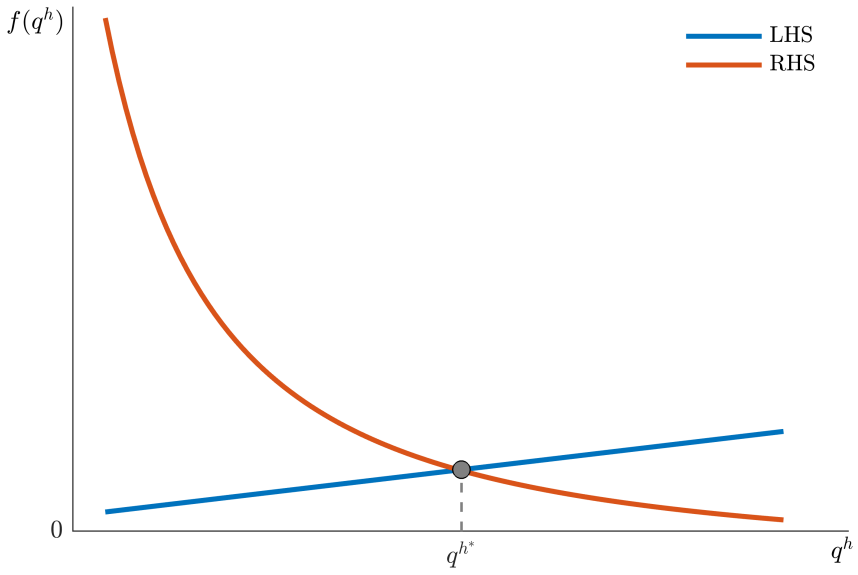


Fig. 2. Equilibrium for the arrival rate of innovation q^h

From (43), a higher intermediate-good subsidy s^x increases q^{h^*} by shifting the orange line to the right. Intuitively, an increase in s^x raises the demand for intermediate goods to amplify automated firms' profits, which thereby simulates more investment on automation.

As for the impact of s^v on the equilibrium arrival rate of automation q^{h^*}, a higher s^v rotates

the blue line up, leading to a lower q^{h^*}. Intuitively, an increase in s^v generates two opposite effects. On the one hand, it increases the investment in vertical innovation, which in turn crowds out resources allocated to automation. On the other hand, the increased investment in vertical innovation raises the market value of vertical innovation in addition to automation, which thereby incentivizes more investment in automation. However, the negative resource crowding-out effect dominates, leading to a decline in the economy's investment in horizontal innovation. As a result, R&D subsidy rate s^v has a negative effect on automation arrival rate q^{h^*}.

In addition, a higher rate of automation subsidy s^h rotates the blue line down and moves the orange line to the right simultaneously. As a result, the equilibrium arrival rate of automation q^{h^*} goes up. The intuition is as follows. An increase in s^h stimulates the incentive for horizontal innovation, raising the economy's investment in automation. Hence, equation (23) implies that the economy exhibits a higher arrival rate of successful automation q^{h^*}.

Finally, equation (41) indicates that the growth rate of income per capita solely depends on the arrival rate of automation when all technical advances are embodied in capital. Therefore, we can obtain the following results.

Proposition 1. *A higher intermediate-good subsidy s^x increases the rate of vertical innovation q^{v^*} and the rate of horizontal innovation q^{h^*}, thereby leading to a higher economic growth rate g. A higher R&D subsidy s^v raises q^{v^*} and reduces q^{h^*}, leading to a lower g. A higher automation subsidy s^h does not affect q^{v^*}, but increases q^{h^*} and g.*

Proof. Proven in the text. □

The above results stand in stark contrast to Chu *et al.* (2023), in which a higher R&D subsidy rate has a positive effect on economic growth and a higher automation subsidy rate has a negative effect on it. In their growth framework with labor-augmenting technical progress, economic growth is driven by innovation, which is based on the traditional Schumpeterian growth paradigm, and capital intensity is endogenously determined by the interaction between innovation and automation. In this context, R&D subsidy encourages innovation and thus stimulates economic growth, whereas the automation subsidy crowds out R&D labor, which in turn dampens the economic growth. By contrast, in our model where technological progress is purely capital-embodied, automation is the sole growth engine, and vertical innovation serves to neutralize capital intensity instead of driving economic growth. As a result, the effects of subsidies are reversed in our model: R&D subsidies reduce economic growth by reallocating resources away from automation, whereas automation subsidies enhance economic growth by accelerating the arrival rate of automation.[16]

[16] R&D subsidies do not necessarily raise long-run growth; they can lower it when they reallocate innovative effort toward a less growth-enhancing margin. Segerstrom (2000) shows that R&D subsidies can reduce long-run growth by reallocating resources toward vertical innovation when horizontal innovation is the stronger engine of growth. Cao *et al.* (2024) also show that innovation subsidies may reduce growth when quantity-based subsidies shift R&D toward lower-quality incremental innovations. Some empirical evidence is consistent with these theoretical results, such as Karhunen and Huovari (2015) and Catozzella and Vivarelli (2016) documenting adverse productivity responses to R&D subsidies.

5.2 Labor share of income

We now analyze the effects of the subsidy instruments on the labor share of income. Combining (33) and (40), we can derive the equilibrium labor share of income such that

$$S_L = 1 - [(1-s^x)\mu]^{\frac{\sigma}{\sigma-1}} \frac{1}{h\phi} \left(\frac{1-s^h}{1-s^v}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}. \tag{44}$$

Based on this equation, we first explore the impact of intermediate-good subsidy s^x on S_L. It is evident that a higher s^x has a positive effect on the labor share of income S_L. The intuition is as follows. A higher s^x stimulates the demand for intermediate goods and expands final-good production. As the demand for labor-based intermediate goods increases, firms require more labor, which pushes up wages. Since the labor-based intermediate goods are competitive, their price rises with the wage rate. By contrast, the price of capital-based intermediate goods remains unchanged. As a result, the relative price of labor-based to capital-based intermediate goods rises. Recall that the elasticity of substitution between intermediate goods is less than unity, the rise in the relative price raises the expenditure share of labor-based intermediates, which are produced solely with labor. Thus, the labor share of income increases.

Next, we proceed to investigate the impacts of R&D and automation subsidies s^v and s^h on S_L, respectively. From equation (44), we can see that S_L is increasing in s^v. Intuitively, a higher R&D subsidy rate s^v makes innovation more attractive relative to automation. As a result, more resources are shifted to vertical innovation, which leads to a higher arrival rate of vertical innovation q^{v^*}. Given that the elasticity of substitution between capital and labor is less than one, advancing the productivity of capital inputs leads to a decline in the capital share, which in turn raises the labor share of income. In contrast, equation (44) indicates that S_L is decreasing in s^h. That is because a higher s^h raises the incentives for automation, so more resources are reallocated toward horizontal-innovation activities; namely expenditures on automation tend to rise. Hence, the arrival rate of successful automation q^h rises, which in turn decreases the labor share because more labor can be replaced by capital. Proposition 2 summarizes these results accordingly.

Proposition 2. *The equilibrium labor share of income S_L increases with intermediate-good subsidy s^x and R&D subsidy s^v, whereas decreases with automation subsidy s^h.*

Proof. Proven in the text □

5.3 Welfare

We now examine the welfare effects of distinct subsidies along the BGP. The consumption level is given by $c_t = c_0 e^{gt}$, where c_0 is the initial consumption level and g is the steady-state growth rate of consumption. Imposing the balanced-growth level of consumption in the households' lifetime utility function (1), we can derive the steady-state welfare such that

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho}\right). \tag{45}$$

Substituting (22), (27), (29) and (31) into the resource constraint $Y_t = c_t L_t + I_t + D_t^v + D_t^h$ yields

$$c_0 = \left[1 - \left(\frac{q^v}{\xi^v}\right)^{\frac{1}{\alpha}} \kappa - \left(\frac{q^h}{\xi^h}\right)^{\frac{1}{\alpha}} h^{-1} - (\mu(1-s^x))^{\frac{1}{\sigma-1}} \kappa\right] \frac{VA(1-\beta_0)^{\frac{1-\sigma}{\sigma}}}{S_L^{1/\sigma}}.$$

Dropping the exogenous term β_0, the steady-state welfare function can be reexpressed as

$$U = \frac{1}{\rho}\left\{\ln\left[1 - \left(\frac{q^v}{\xi^v}\right)^{\frac{1}{\alpha}} \kappa - \left(\frac{q^h}{\xi^h}\right)^{\frac{1}{\alpha}} h^{-1} - (\mu(1-s^x))^{\frac{1}{\sigma-1}} \kappa\right] - \frac{1}{\sigma}\ln S_L + \frac{\sigma-1}{\sigma\rho} q^h\right\}. \tag{46}$$

In this welfare function, the first two terms determine the welfare arising from the initial consumption level. Specifically, the first term represents the share of output allocated to consumption, while the second term captures the initial level of output. The third term reflects the welfare arising from balanced growth rate. Based on this specification, we analyze the welfare implications of various subsidies, namely subsidies to intermediate goods, R&D, and automation, respectively.

First, we examine the welfare effect of intermediate-good subsidy s_x. From Proposition 1, an increase in s_x raises the arrival rates of both innovation and automation (q^h and q^h). This, in turn, reduces the resources devoted to consumption while stimulates long-run economic growth. Furthermore, as indicated in Proposition 2, a higher s_x raises the labor share of income S_L, which further increases the initial level of output. Therefore, the welfare effect of intermediate good subsidy is determined by these three counteracting forces. If the positive welfare effects from higher long-run growth and a higher initial output level dominate the negative effect from reduced initial consumption share, a higher s_x generates a positive effect on the steady-state welfare; otherwise, it generates a negative effect on the steady-state welfare.

Next, we turn to the welfare effect of automation subsidy s_h. On the one hand, a higher s_h accelerates economic growth, which contributes positively to welfare through the third term of equation (46). On the other hand, it generates a negative welfare effect by reducing the initial consumption level. This reduction operates through two channels: it decreases both the share of output devoted to consumption (the first term) and the initial level of output (the second term). Intuitively, the use of automation subsidies incentivizes more resources allocated to horizontal innovation, raising the arrival rate of automation q^h. A higher q^h leads to more industries engaging in vertical innovation and more labor being replaced by capital. Thus, the resources devoted to consumption falls. At the same time, the labor share of income S_L declines, indicating a lower level of output. Overall, whether automation subsidies promote or dampen the steady-state welfare depends on the interplay between the positive impact from growth and the negative impact from consumption.

Finally, we analyze the welfare effect of R&D subsidy s_v. Proposition 1 and Proposition 2 indicate that an increase in s_v generates two opposing effects on the components of welfare. Specifically, an increase in s_v harms steady-state welfare because it hinders economic growth. However, it improves welfare by raising the initial output level. In addition, the effect of s_v on the

consumption share of output (the first term of (46)) becomes ambiguous. This is because although a higher s_v reallocates resources to vertical innovation, but it simultaneously reduces resources allocated to automation and capital production. Given these effects of s_v on the components of steady-state welfare, the overall welfare effect of R&D subsidy remains ambiguous.

In summary, due to the complexity of the steady-state welfare function, it is analytically challenging to determine the net welfare effect of each single subsidy. Therefore, we evaluate the effects of these subsidies on steady-state welfare through the quantitative analysis, which are present in the following section.

6 Quantitative Analysis

In this section, we first calibrate the model to aggregate US data to perform a quantitative analysis. Then we numerically evaluate the effects of subsidies on growth, labor share, and steady-state welfare, respectively.

6.1 Calibration

The model features the following set of parameters $\{\rho, \sigma, v, \psi, \alpha, \phi, \xi^h, \xi^v, h, \mu, s^x, s^v, s^h\}$. We choose a conventional value of 0.04 for the discount rate ρ. Without loss of generality, we normalize $v = 1$. The marginal cost of capital input is thereby set to $\psi = v = 1$. As for patent breadth, we follow the empirical range reported in Jones and Williams (2000) (i.e., 1.05–1.4) and choose a modest value $\mu = 1.2$ as the markup ratio. Following Impullitti (2010), we set the diminishing returns on R&D α to 0.4. In addition, given that the US has not used manufacturing subsidies to the purchase of intermediate goods in the past three decades, we follow Hu *et al.* (2023) to set $s^x = 0$. As for R&D subsidy, we follow Grossmann *et al.* (2013) to set the rate in the US to 0.066. Then, we follow Chu *et al.* (2023) to consider a symmetric subsidies $s^h = s^v = 0.066$ as our benchmark, since the US currently does not distinguish between subsidies for innovation and automation.[17]

Next, we calibrate the substitution parameter σ using the elasticity of substitution between capital and labor in the US, which is pinned down as 0.6.[18] As for the R&D productivity parameter, we calibrate the value of ξ^v by matching the arrival rate of vertical innovation. Following Lanjouw (1998), who estimates the probability of obsolescence ranges between 7% and 12%, we choose a modest value of 10% as the standard moment. This value implies an average duration of 10 years between successive vertical innovations. Given the calibrated value of ξ^v, we then calibrate the productivity parameter ξ^h by targeting the empirical long-run growth rate of GDP

[17] According to the US Internal Revenue Code, eligibility for the research tax credit depends only on whether an activity meets the statutory definition of qualified research; the code does not differentiate between automation-oriented research and other forms of R&D, nor does it assign separate credit rates to them. Therefore, we follow Chu *et al.* (2023) to use symmetric subsidies as a benchmark and then vary the individual subsidy rates s^h and s^v separately to examine their differential effects.

[18] Oberfield and Raval (2021) provide an estimate for the aggregate elasticity of substitution between capital and labor for the manufacturing sector, which lies between 0.5 to 0.7, and we choose the intermediate value. In Appendix A.3, we use 0.5 to conduct a sensitivity analysis.

per capita in the US at 1.5%, which is consistent with the value used in Chu *et al.* (2012). Finally, to calibrate the magnitude of h, we follow Yang (2018) to set the step size $1+\phi$ as 1.09 and use the US labor income share of 0.6.[19] The calibrated parameter values are summarized in Table 1.

Table 1: Benchmark Calibration Summary

Parameters	Description	Value	Source/Target
External			
ρ	Discount rate	0.04	Traditional value
v	Factor productivity	1	Normalized
ψ	Marginal cost of capital	1	$\psi = v$
μ	Patent breath	1.2	Jones and Williams (2000)
α	Diminishing returns on R&D	0.4	Impullitti (2010)
ϕ	Step size	0.09	Yang (2018)
Internal			
s^x	Intermediate-good subsidy	0	Intermediate-good subsidy rate
s^v	Subsidy on innovation	0.066	The US R&D subsidy rate
s^h	Subsidy on automation	0.066	Chu *et al.* (2023): $s^h = s^v$
σ	Substitution parameter	$-2/3$	EoS between labor and capital (0.6)
ξ^v	R&D productivity parameter	0.221	Probability of obsolescence (10%)
ξ^h	Automation productivity	0.041	Economic growth rate (1.5%)
h	Initial productivity parameter	1.793	Labor share of income (0.6)

6.2 Benchmark results

Based on the benchmark calibration, we examine the individual effects of intermediate-good subsidy s^x, R&D subsidy s^v, and automation subsidy s^h on the economic growth rate, the labor share of income, and steady-state welfare in this subsection.

Figure 3 depicts the effects of subsidies to intermediate goods, exhibiting positive effects both on the economic growth g and the labor share of income S_L. Specifically, increasing s^x from 0% to 20% raises the economic growth rate from 1.50% to 1.59%, and lifts the labor share from 60.00% to 63.42%. Moreover, s_x exhibits an inverted-U shaped effect on the steady-state welfare. When $s^x \in (0, 0.16)$, the welfare gains from higher growth and an increased initial output level

[19] According to data from PWT10, the average labor share of income in the US between 2009 and 2019 is approximately 60%.

outweigh the loss due to a reduced consumption share. However, once s^x exceeds 16%, the negative welfare effect becomes dominant, leading to an overall decline in welfare.

The effects of vertical innovation subsidy s^v on g, S_L and ΔU are presented in Figure 4. It can be seen that R&D subsidy s^v negatively impacts economic growth but benefits the labor share of income. For example, an increase in s^v from 0 to 20% decreases the economic growth rate by 0.07 percentage points but raises the labor share of income by roughly 6 percentage points, reaching 63.92%. In terms of welfare, R&D subsidy yields a positive net effect, suggesting that the growth-reducing welfare impact is dominated by other beneficial forces in this case.

In contrast to R&D subsidy, automation subsidy s_h has diametrically opposite effects on these variables of interest. As shown in Figure 5, a higher automation subsidy s_h stimulates economic growth, while it adversely affects the labor share of income and steady-state welfare. A similar increase in s^h from 0% to 20% raises the growth rate from 1.43% to 1.66%, while it reduces the labor share from 61.78% to 55.65%. Welfare gain ΔU is also negatively correlated with s^h, implying that the positive growth-welfare effect is dominated by the negative consumption-welfare effect. These results are in line with the implications of Proposition 1 and Proposition 2.

In summary, these three subsidies have markedly different implications for growth, the labor share, and welfare. The intermediate-good subsidy delivers a dual benefit by promoting both economic growth and the labor share, with welfare exhibiting an inverted-U pattern that suggests an optimal subsidy rate of around 16%. The use of R&D subsidy involves a tradeoff: it enhances steady-state welfare and raises the labor share, while dampening economic growth. By contrast, the automation subsidy stimulates economic growth at the expense of labor share and steady-state welfare. These results highlight the importance of balancing the objectives of growth, income distribution, and welfare in the design of subsidy policies under a framework with capital-bodied technical change.

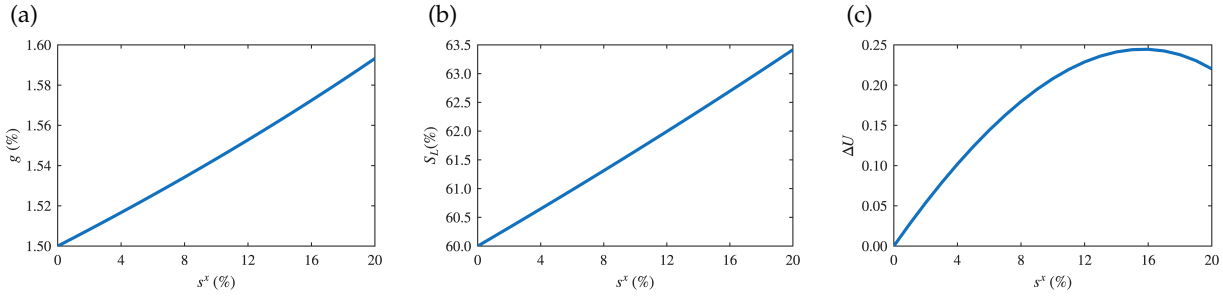


Fig. 3. Effect of s^x on g, S_L and ΔU.

To further assess the interaction between automation and R&D subsidies, we make a comparison across four cases: (1) the benchmark case where realistic values are calibrated (i.e., $s^x = 0, s^v = s^h = 0.066$); (2) the unbalanced case with a higher automation subsidy rate where s^h is increased proportionally by 0.1 (i.e., $s^x = 0, s^v = 0.066, s^h = 0.166$); (3) the unbalanced case with a higher R&D subsidy rate where s^v is increased proportionally by 0.1 (i.e., $s^x = 0, s^v = 0.166, s^h = 0.066$); (4) the balanced case where s_h and s_v are both increased proportionally by 0.1 (i.e., $s^x = 0, s^v = 0.166, s^h = 0.166$). The key variables for these cases are

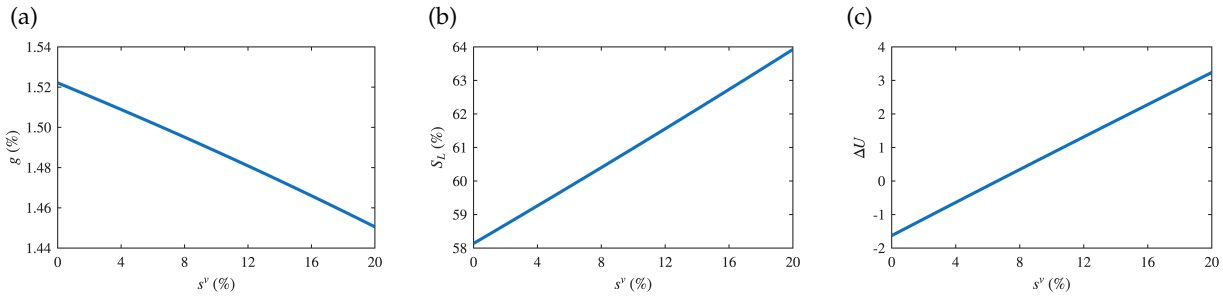


Fig. 4. Effect of s^v on g, S_L and ΔU.

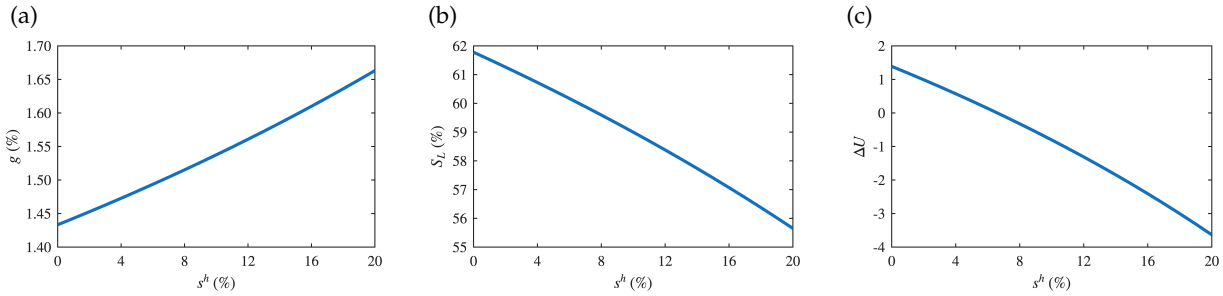


Fig. 5. Effect of s^h on g, S_L and ΔU.

summarized in Table 2.

Table 2: Comparisons among different scenarios

Scenarios	q^h	q^v	g	S_L	ΔU
(i) Benchmark case ($s^x = 0, s^v = s^h = 0.066$)	0.0060	0.100	1.50%	60.00%	–
(ii) Higher automation subsidy ($s^x = 0, s^v = 0.066, s^h = 0.166$)	0.0065	0.100	1.62%	56.86%	-2.5838
(iii) Higher R&D subsidy ($s^x = 0, s^v = 0.166, s^h = 0.066$)	0.0059	0.105	1.46%	62.91%	2.4243
(iv) Balanced case ($s^x = 0, s^v = 0.166, s^h = 0.166$)	0.0063	0.105	1.58%	60.00%	0.1795

Scenarios (ii) and (iii) broadly confirm the policy implications of automation subsidy and R&D subsidy discussed above. Increasing s_h by 0.1 raises the arrival rate of automation q^h and accelerates economic growth from 1.50% to 1.62%. However, this modest growth gain is accompanied by a sharp decline in the labor share, from 60% to 56.86%, and a welfare loss of 2.58. By contrast, increasing s_v by 0.1 raises the arrival rate of vertical innovation q^v and increases the labor share to 62.91%, generating a welfare gain of 2.42, although economic growth declines slightly to 1.46%.

From scenario (iv), we can see that introducing balanced subsidies for both automation and

vertical innovation leads to higher arrival rates for both types of innovation, leading to a higher economic growth rate, which is primarily driven by q^h. Furthermore, while automation subsidy tends to reduce the labor share of income, this negative effect is completely offset by the positive impact of R&D subsidy. Consequently, the labor share remains unchanged. This result occurs because the ratio of q^h to q^v remains unchanged when both subsidies are set at the same rate, as shown in equation (39). In this case, the positive value of ΔU indicates that this combined subsidy policy gives rise to welfare gains. Compared to scenario (ii) and (iii), the balanced policy mix of automation and R&D subsidies enhances economic growth and social welfare without distorting the labor share of income.

In summary, while automation subsidy can stimulate growth, it comes at a cost to labor share and welfare. Conversely, raising R&D subsidy increases both labor share and welfare, but hinders economic growth. These tradeoffs highlight the potential complementarity of automation and R&D subsidies in policy design. The balanced case, in which subsidies on both automation and vertical innovation are adjusted cooperatively, achieves moderate growth and welfare gains while preserving the labor share of income. Therefore, a balanced mix of automation subsidy and R&D subsidy can offer a reasonable compromise among competing objectives of growth, income distribution, and welfare.

7 Transition dynamics

So far, we have focused on the long-run implications of the subsidies. In this section, we use the relaxation algorithm in Trimborn *et al.* (2008) to simulate the transition paths of key variables and examine the short-run effects of each subsidy. Before doing this, we first derive the system that characterizes the dynamics of this macroeconomy with capital-embodied technical change.

7.1 The dynamical system

Define transformed variables $\chi_t^h \equiv D_t^h/Y_t$ and $\chi_t^v \equiv D_t^v/Y_t$, which are referred to as expenditure share of horizontal innovation and expenditure share of vertical innovation, respectively. Moreover, define the capital intensity by $\kappa_t \equiv \int_0^{\beta_t} [z_t(i)]^{-1} di = \beta_t Z_t^{-1}$. From equation (22), the arrival rate of vertical innovation q_t^v can be written as

$$q_t^v = \left(\frac{\chi_t^v}{\kappa_t}\right)^{\alpha} \xi^v. \tag{47}$$

Hence, we have $\dot{q}_t^v/q_t^v = \alpha(\dot{\chi}_t^v/\chi_t^v - \dot{\kappa}_t/\kappa_t)$. Given the quality $z_t(i)$ for the monopolist in industries $i \in (0, \beta_t)$, equation (21) indicates that $\dot{V}_t/V_t = (1/\alpha - 1)\dot{q}_t^v/q_t^v + \dot{Y}_t/Y_t$. Then using (16) and (21), we can rewrite the no-arbitrage condition (18) as:

$$r_t - \frac{\dot{Y}_t}{Y_t} - \left(\frac{1-\alpha}{\alpha}\right)\frac{\dot{q}_t^v}{q_t^v} = \frac{(1-s^x)^{\frac{1}{\sigma-1}}\Theta(\xi^v)^{\frac{1}{\alpha}}}{1-s^v}\left(q_t^v\right)^{1-\frac{1}{\alpha}} - q_t^v. \tag{48}$$

Next, substituting (22), (27), (29) and (31) into the resource constraint $Y_t = c_t L_t + I_t + D_t^v + D_t^h$, we have

$$\frac{Lc_t}{Y_t} = 1 - \chi_t^v - \chi_t^h - [\mu(1-s^x)]^{\frac{1}{\sigma-1}}\kappa_t, \tag{49}$$

which implies that $\dot{c}_t/c_t - \dot{Y}_t/Y_t = -[\dot{\chi}_t^v + \dot{\chi}_t^h + (\mu(1-s_x))^{\frac{1}{\sigma-1}}\dot{\kappa}_t]/[1-\chi_t^v-\chi_t^h-(\mu(1-s_x))^{\frac{1}{\sigma-1}}\kappa_t]$. Combined with Euler equation (3) and (47), equation (48) can be reexpressed as:

$$\rho - \frac{\dot{\chi}_t^v + \dot{\chi}_t^h + M\dot{\kappa}_t}{\chi_t^c} - (1-\alpha)\left(\frac{\dot{\chi}_t^v}{\chi_t^v} - \frac{\dot{\kappa}_t}{\kappa_t}\right) = \frac{(1-s^x)^{\frac{1}{\sigma-1}}\Theta\xi^v}{1-s^v}\left(\frac{\chi_t^v}{\kappa_t}\right)^{(\alpha-1)} - \left(\frac{\chi_t^v}{\kappa_t}\right)^{\alpha}\xi^v, \tag{50}$$

where $M \equiv [\mu(1-s_x)]^{\frac{1}{\sigma-1}}$, and $\chi_t^c \equiv 1 - \chi_t^v - \chi_t^h - M\kappa_t$ captures the expenditure share on consumption.

Furthermore, equation (22) indicates that the arrival rate of automation q_t^h is given by

$$q_t^h = (\chi_t^h h)^{\alpha}\xi^h, \tag{51}$$

implying that $\dot{q}_t^h/q_t^h = \alpha(\dot{\chi}_t^h/\chi_t^h)$. Recall that once automation is successful, the newly-automated industry enters a vertical innovation phase, where the present value of this new innovation $V_t(i)$ will then be again given by (18). Substituting (16) and (26) into the no-arbitrage condition (18), we have

$$r_t - \frac{\dot{Y}_t}{Y_t} - \frac{1-\alpha}{\alpha}\frac{\dot{q}_t^h}{q_t^h} = \frac{(1-s^x)^{\frac{1}{\sigma-1}}\Theta(\xi^h)^{\frac{1}{\alpha}}}{1-s^h}\left(q_t^h\right)^{1-\frac{1}{\alpha}} - q_t^v. \tag{52}$$

With Euler equation (3) and (51), this equation can be written as:

$$\rho - \frac{\dot{\chi}_t^v + \dot{\chi}_t^h + M\dot{\kappa}_t}{\chi_t^c} - (1-\alpha)\frac{\dot{\chi}_t^h}{\chi_t^h} = \frac{(1-s^x)^{\frac{1}{\sigma-1}}\Theta\xi^h}{1-s^h}\left(\chi_t^h h\right)^{(\alpha-1)} - \left(\frac{\chi_t^v}{\kappa_t}\right)^{\alpha}\xi^v. \tag{53}$$

In addition, differentiating $\kappa_t = \int_0^{\beta_t}\frac{1}{z_t(i)}di$ by applying Leibniz's rule, we can obtain:

$$\dot{\kappa}_t = -\int_0^{\beta_t}\frac{\dot{z}_t(i)}{z_t(i)^2}di + \frac{\dot{\beta}_t}{z_t(\beta_t)}. \tag{54}$$

In the vertical innovation process, a given automated industry raises its productivity proportionally by an amount of ϕ, leading to an expected growth rate of $\dot{z}_t/z_t = \phi q^v$. At the same time, the automation frontier expands at the rate of $\dot{\beta}_t = (1-\beta_t)q_t^h$. Incorporating these specifications, together with (47) and (51), the law of motion for κ_t can be eventually reduced to

$$\dot{\kappa}_t = (\chi_t^h)^{\alpha}h^{(\alpha-1)}\xi^h - \phi\left(\frac{\chi_t^v}{\kappa_t}\right)^{\alpha}\xi^v\kappa_t. \tag{55}$$

By now, we have obtained three endogenous variables $\{\chi_t^h, \chi_t^v, \kappa_t\}$ and their relations, given by (50), (53) and (55), which will determine the dynamical system of this economy. Then we

further derive the law of motion for χ_t^h and χ_t^v. Combining (50) and (53) yields

$$\dot{\chi}_t^v = \frac{\chi_t^v}{\kappa_t}\dot{\kappa}_t + \frac{\chi_t^v}{\chi_t^h}\dot{\chi}_t^h - \frac{\chi_t^v}{1-\alpha}\left[\xi^v R_v\left(\frac{\chi_t^v}{\kappa_t}\right)^{\alpha-1} - \xi^h R_h(\chi^h h)^{\alpha-1}\right], \tag{56}$$

where $R_v \equiv (1-s^x)^{\frac{1}{\sigma-1}}\Theta/(1-s^v)$, and $R_h \equiv (1-s^x)^{\frac{1}{\sigma-1}}\Theta/(1-s^h)$. Substituting (55) and (56) into (53), the law of motion for χ_t^h is given by

$$\dot{\chi}_t^h = \frac{1}{\Gamma_t^0}\left[\rho + \Gamma_t^1\xi^v\left(\frac{\chi_t^v}{\kappa_t}\right)^{\alpha} - \Gamma_t^2\xi^h(\chi_t^h h)^{\alpha-1}\right], \tag{57}$$

where $\Gamma_t^0 \equiv (1+\chi_t^v/\chi_t^h)/\chi_t^c + (1-\alpha)/\chi_t^h$, $\Gamma_t^1 \equiv 1 + \{[R_v/(1-\alpha) + M\phi]\kappa_t + \phi\chi_t^v\}/\chi_t^c$, and $\Gamma_t^2 \equiv R_h\{1+\chi_t^v/[\chi_t^c(1-\alpha)]\} + (\chi_t^h/\chi_t^c)(\chi_t^v/\kappa_t + M)$. Finally, substituting the law of motion for χ_t^h (57) and the law of motion for κ_t (55) into (56) yields the law of motion for χ_t^v such that

$$\dot{\chi}_t^v = \chi_t^v\left\{\frac{\rho}{\chi_t^h\Gamma_t^0} + \xi^v\left(\frac{\chi_t^v}{\kappa_t}\right)^{\alpha}\left[\frac{\Gamma_t^1}{\Gamma_t^0\chi_t^h} - \phi - \frac{R_v\kappa_t}{(1-\alpha)\chi_t^v}\right] + \xi^h(\chi_t^h h)^{\alpha-1}\left(\frac{\chi_t^h}{\kappa_t} + \frac{R_h}{1-\alpha} - \frac{\Gamma_t^2}{\Gamma_t^0\chi_t^h}\right)\right\}. \tag{58}$$

In summary, equations (55), (57) and (58) characterize the dynamic system of this economy. According to these three differential equations, we can apply relaxation algorithm to simulate the transitional-dynamic effects of intermediate good subsidy s^x, R&D subsidy s^v, and automation subsidy s^h, respectively.

7.2 Dynamic policy implications

Figure 6 depicts the dynamic effects of intermediate-good subsidy on interested variables, indicating that an increase in s_x generates no transitional dynamics. Figure 6a shows that a higher s_x does not affect the state variable κ. However, the control variables χ^v and χ^h immediately jump to their new steady-state values, as shown in Figure 6b and 6c. As a result, the expenditure share on consumption χ^c falls from 61.11% to 58.52% (see Figure 6d). The dynamic responses of economic growth g and the labor share S_L to an increase in s_x, presented in Figures 6e and 6f, are consistent with its long-run effects. Specifically, raising the intermediate-good subsidy increases both economic growth and the labor share of income.

Figure 7 illustrates the dynamic effects of increasing automation subsidy from 0.066 to 0.166. As shown in Figure 7c, a higher automation subsidy leads to an immediate and sharp increase in the expenditure share on automation. Although this share gradually declines toward the new steady state, it remains above its initial level. The initial surge in χ^h accelerates the arrival rate of automation, raising the growth rate of β_t. The resulting imbalance between the growth rates of β_t and Z_t increases the ratio of the technology indices $\beta_t Z_t^{-1}$. As more industries are automated, the growth rate of β_t slows down until it aligns with that of Z_t. In the new steady state, the economy reaches a higher level of $\beta_t Z_t^{-1}$, as depicted in Figure 7a. Figure 7b shows that the expenditure share on vertical innovation experiences a slight initial increase, as the higher arrival rate of automation raises firms' probability of engaging in vertical innovation, thereby attracting

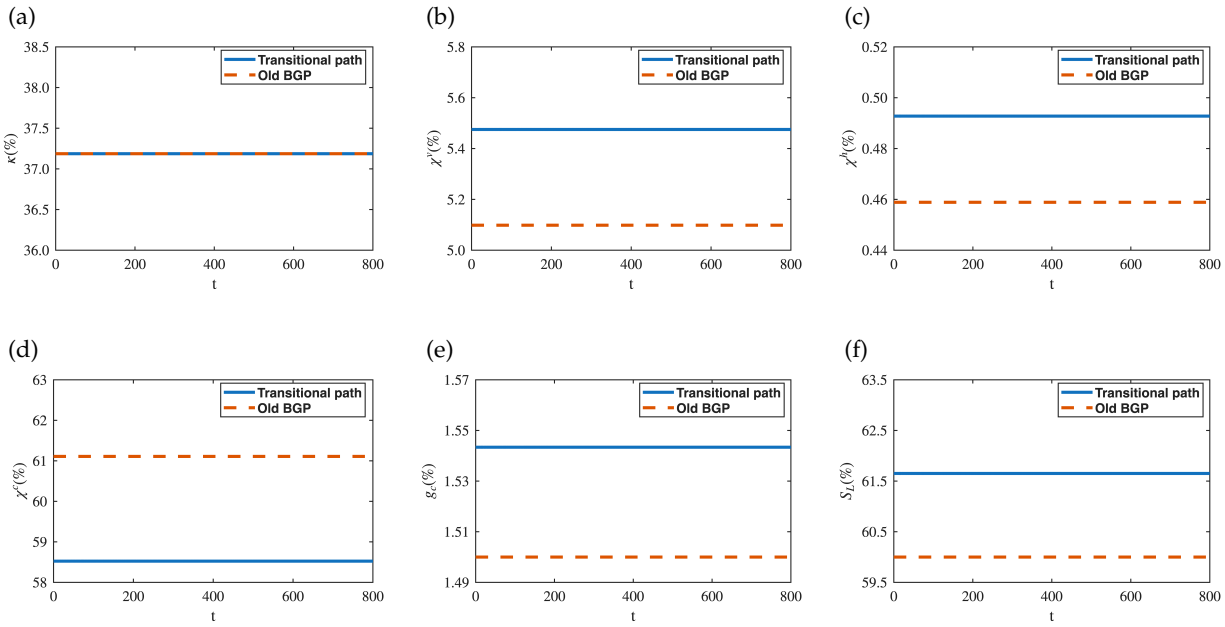


Fig. 6. Dynamic effects of s^x on κ, χ^v, χ^h, χ^c, g_c, S_L.

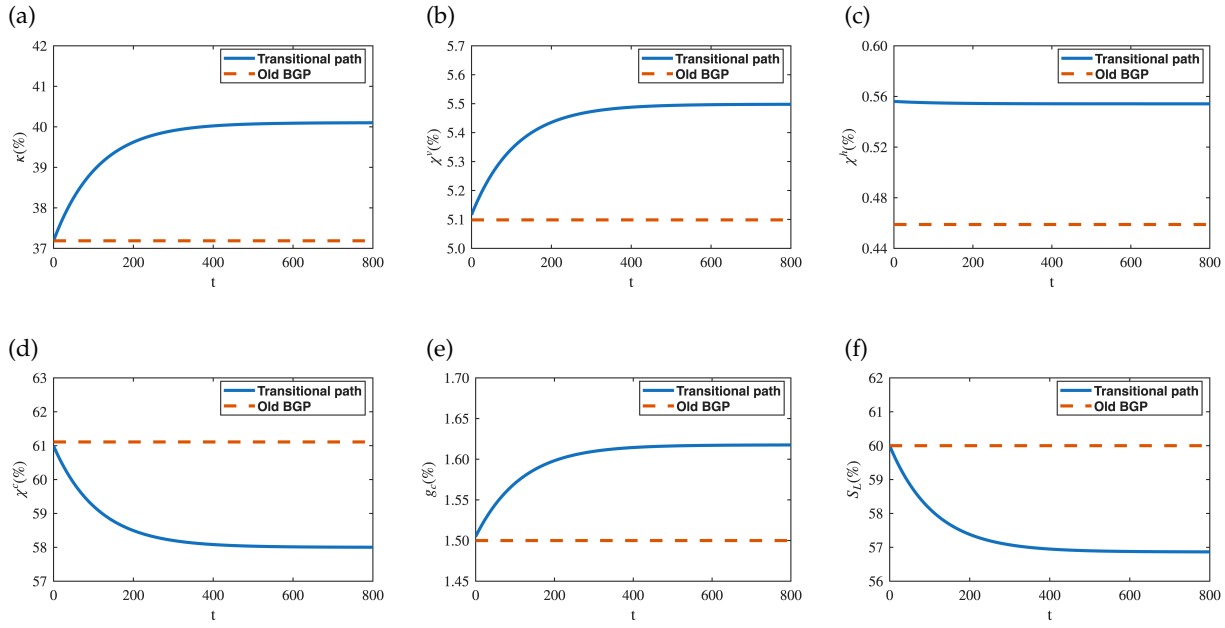


Fig. 7. Dynamic effects of s^h on κ, χ^v, χ^h, χ^c, g_c, S_L.

more resources to invest in vertical innovation. Subsequently, χ^v gradually converges to a higher steady-state level. In contrast, Figure 7d shows that resources allocated to consumption slightly fall at the beginning and then gradually drop to 58%. Moreover, increases in χ^h and χ^v accelerate automation and capital productivity growth, respectively. These two forces act simultaneously to raise the growth rate along the transition path, pushing it toward a higher level. On the new BGP, the higher long-run growth rate is sustained primarily by a higher arrival rate of automation. Figure 7f illustrates the response of the labor share of income, which gradually falls to the new steady-state level.

The transitional dynamic effects of raising R&D subsidy by 0.1 are summarized in Figure 8. Figure 8b shows that increasing s^v from 0.066 to 0.166 initially raises the expenditure share on vertical innovation. Although this share subsequently declines, it converges to a new steady-state level that is above its initial level. At the same time, the increase in s^v diverts resources away from horizontal innovation. Accordingly, the expenditure share on automation initially falls below the original steady-state level and gradually converges to the new counterpart (see Figure 8c). The rise in s^v also raises the arrival rate of vertical innovation, which accelerates the growth rate of Z_t. This temporarily creates an imbalance between the growth rates of Z_t and β_t, decreasing the technology ratio $\kappa \equiv \beta_t Z_t^{-1}$. Along the transition, the economy converges to a new steady state by rebalancing the growth rates of Z_t and β_t. Figure 8a shows that κ gradually stabilizes at a permanently lower steady-state level. The reallocation of resources also affects consumption dynamics. As depicted in Figure 8d, the consumption share initially falls below its benchmark level, but it gradually rebounds and eventually exceeds it in the new steady state. In contrast, the growth rate of consumption exhibits a distinctive pattern (see Figure 8e). Following the increase in s^v, the growth rate g_c jumps to approximately 1.57%, indicating a short-run boost.

Although a decline in χ^h slows the expansion of automation, the increase in χ^v raises the arrival rate of vertical innovation and accelerates productivity growth in capital inputs, generating an additional source of growth along the transition path. This positive effect of faster capital productivity growth dominates the negative effect of slower automation, so that the vertical R&D subsidy temporarily raises the growth rate. However, this effect is transitory: as the growth rates of Z_t and β_t are gradually rebalanced and the economy converges to the new steady state, the additional transitional growth effect arising from the faster growth of Z_t dissipates. Once the transitional advantage disappears, the slowdown in automation then solely governs the long-run growth, leading g_c to converge to a lower long-run level. Finally, Figure 8f shows that a higher s^v gradually leads the labor share to rise along the transition path until it reaches a new steady state. This is driven by the fact that in the presence of a less-than-unitary elasticity of substitution between the two factors, advancing capital productivity reduces the capital share, and thus increases the labor share.

7.3 Welfare effects along the transition

Having examined how each subsidy instrument affects the dynamics of consumption share and growth rate, we now turn to their welfare implications. Due to the complexity of transitional dynamics, the majority of existing literature evaluates policy welfare effects mainly through

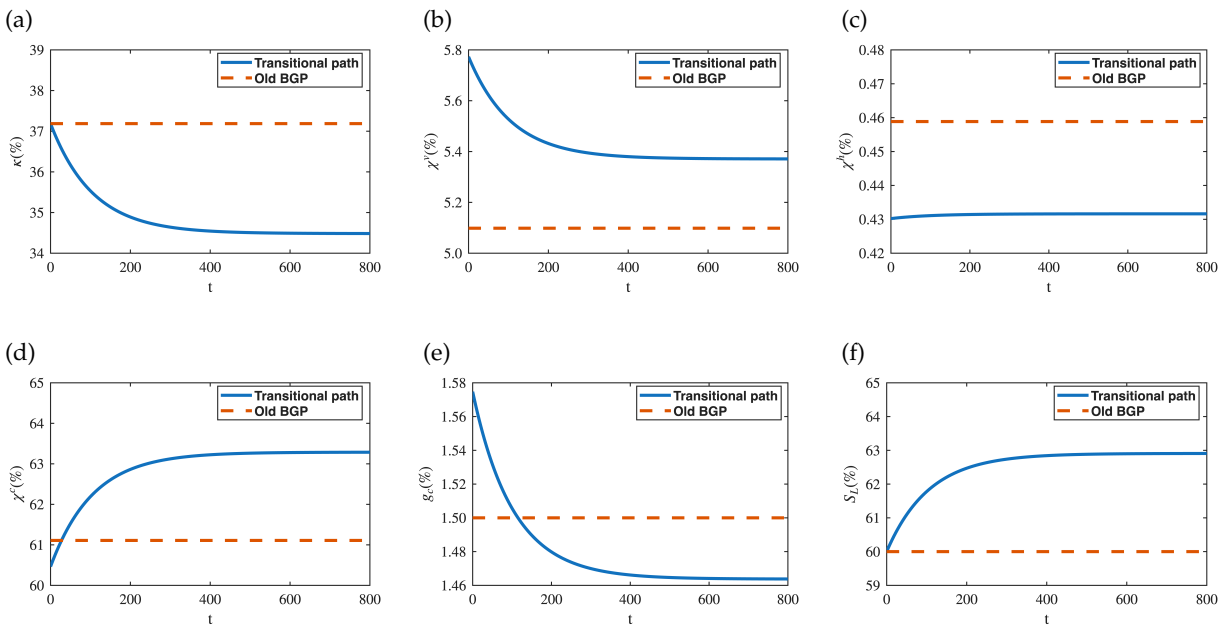


Fig. 8. Dynamic effects of s^v on κ, χ^v, χ^h, χ^c, g_c, S_L.

steady-state comparisons. However, neglecting transition dynamics may lead to biased welfare assessments, potentially overestimating or underestimating the true benefits of policy interventions, and thereby misleading policymakers. To provide a full welfare analysis, in this subsection we quantify the transitional welfare effects of the three subsidies and compare them with their steady-state counterparts.

Figure 9 illustrates the transitional welfare effects of the intermediate goods subsidy s^x, the automation subsidy s^h and the R&D subsidy s^v, respectively. Recalling that intermediate good subsidy s^x has no transitional dynamics effects on resource allocations, it is apparent that the transitional welfare effect of intermediate good subsidy is consistent with steady-state welfare, as shown in Figure 9a. Moreover, the relation between transitional welfare and s^x is inverted-U shape, peaking at an optimal rate of 16.6%.

Figure 9b reveals a striking welfare reversal between the steady-state and transitional effects of automation subsidy, highlighting the importance of accounting for transition dynamics. As depicted by the dashed orange line, the steady-state welfare effect declines monotonically with s^h, implying that a higher automation subsidy is detrimental to welfare in the long run. However, the blue line shows that the transitional welfare effect exhibits an inverted-U pattern, reaching its maximum at around 51%. As aforementioned, along the BGP, a higher s^h generates a dominant negative consumption-welfare effect through the sharp decline in the labor share of income S_L and the consumption share of output χ^c. However, the decrease in the initial level of consumption is relatively smaller due to the slight decline in the consumption share of output. Consequently, along the transition path, the welfare losses from a lower initial consumption level may be offset by the welfare gains from accelerated economic growth. Interestingly, discounting and

integrating these early gains over the entire path outweigh the long-term welfare loss, leading to a positive transitional welfare effect over a substantial range of subsidy rates. Policy implications differ sharply due to the welfare reversal: the steady-state welfare evaluation suggests refraining from automation subsidies, whereas the transitional welfare analysis supports a moderate level of automation subsidy. Overall, these findings highlight the importance of considering transitional dynamics in policy design, as steady-state analysis can understate the potential benefits of automation subsidies by overlooking substantial welfare gains during the transition.

In contrast to automation subsidy, the welfare effects of R&D subsidy exhibit a different form of intertemporal tradeoff. As shown in Figure 9c, when the R&D subsidy remains below a threshold of approximately 50%, the transitional welfare effects and the steady-state welfare effects of a higher R&D subsidy are both positive. However, the transitional welfare gain is significantly smaller than its steady-state counterpart. This discrepancy arises because steady-state welfare captures only long-run improvements, whereas transitional welfare also accounts for welfare costs along the transition path. In the long run, an increase in s^v simultaneously raises the labor share S_L and the consumption share of output χ^c such that the initial level of consumption is increased. This thereby generates positive steady-state welfare effect that dominates the negative one arising from the declined economic growth.

By contrast, along the transition path, an increase in s^v reallocates resources toward R&D activities. This shift depresses the initial level of consumption below the original BGP, bringing about a short-run consumption loss despite of the improved growth during the transition. Consequently, when these early consumption losses are taken into account, the overall transitional welfare gains become noticeably smaller than the steady-state welfare gains. Moreover, the transitional welfare peaks at an optimal rate of 33%, beyond which further increases in s^v reduce the transitional welfare gains. Once s^v becomes sufficiently large, the short-run losses may dominate, causing transitional welfare to turn negative, even if the steady-state welfare remains positive. These findings indicate that while steady-state analysis alone might favor a larger R&D subsidy, accounting for transitional dynamics reveals a crucial implication: excessively high R&D subsidy can lead to transitional welfare losses. Therefore, a more modest subsidy rate, such as 33% we identified here, is optimal.

In summary, the analysis of transitional welfare effects yields three key insights. First, for the intermediate-good subsidy, the absence of transitional dynamics ensures its welfare effect is fully captured by steady-state analysis, peaking at the rate of 16.6%. Second, the automation subsidy exhibits a striking welfare reversal: while steady-state analysis suggests that subsidizing automation is detrimental to long-run welfare, incorporating transitional dynamics reveals significant short-run welfare gains that justify a moderate subsidy rate of around 51%. Third, the R&D subsidy reveals a different form of intertemporal tradeoff: although subsidizing vertical innovation is beneficial in the long run, the short-run consumption losses along the transition path limit its optimal rate to 33%. Taken together, relying solely on steady-state analysis may underestimate the benefits of automation subsidies while overestimating the welfare gains from R&D subsidy. These findings underscore the critical importance of accounting for transitional dynamics in policy evaluation.

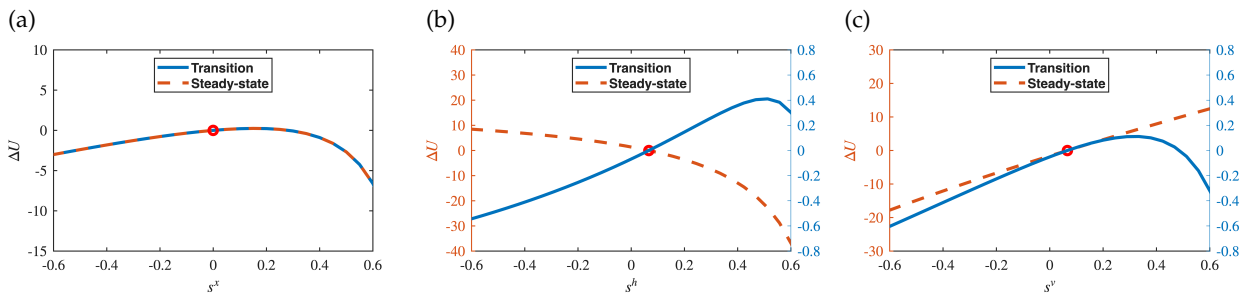


Fig. 9. Transitional welfare effects of s^x, s^h, s^v

7.4 Dynamically optimal policy analysis

The preceding analysis allows us to evaluate the individual effect of each policy instrument. However, we have not yet considered the use of a set of policies and therefore leave an important question unanswered: what is the optimal policy when all instruments can be deployed jointly? To answer this question, we first use relaxation algorithm to characterize the socially optimal allocation that maximizes the welfare of this model economy. Then we further numerically solve for the dynamically optimal policy regime, a time-variant combination of subsidies, to replicate the first-best allocations.

7.4.1 Socially optimal solution

First, the social planner's objective is to maximize the discounted utility of the representative consumer. Given the total resources devoted to innovations, current consumption can be increased at no cost in terms of future consumption if $Y_t - I_t$ is not maximized due to the resource constraint $Y_t = c_t L_t + I_t + D_t^v + D_t^h$. Thus, using (4), the social planner maximizes

$$Y_t - I_t = v\left[\int_0^{\beta_t}\left(z_t(i)^{\frac{\rho}{\rho-1}}x_t(i)\right)^{\rho}di + \int_{\beta_t}^{1}\left(\frac{AL}{1-\beta_t}\right)^{\rho}di\right]^{\frac{1}{\rho}} - \int_0^{\beta_t}\psi x_t(i). \tag{59}$$

Differentiating it with respect to $x_t(i)$ yields $x_t(i) = Y_t/(vz_t(i))$, which implies that

$$I_t = \psi\int_0^{\beta_t}x_t(i)di = \beta_t Z_t^{-1}Y_t. \tag{60}$$

In addition, the demand for the automated good $y_t(i)$ in the social optimum becomes

$$y_t(i) = v^{-1}z_t(i)^{-\frac{1}{\sigma}}Y_t. \tag{61}$$

Substituting (11) and (61) into (4) yields the total output such that

$$Y_t = \frac{vAL\left(1-\beta_t\right)^{\frac{1-\sigma}{\sigma}}}{\left(1-\beta_t Z_t^{-1}\right)^{1/\sigma}}. \tag{62}$$

Define by $\Gamma_t \equiv 1-\beta_t$ the fraction of non-automated industries. Using (27), we obtain the dynamics of automation such that

$$\dot{\Gamma}_t = -\xi^h h^\alpha (\chi_t^h)^\alpha \Gamma_t. \tag{63}$$

Then the total output in the social optimum can be expressed as:

$$Y_t = \frac{vAL\left(\Gamma_t\right)^{\frac{1-\sigma}{\sigma}}}{\left(1-\kappa_t\right)^{1/\sigma}}. \tag{64}$$

Next, the social planner solves the optimal control problem such that

$$\max \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{65}$$

subject to the dynamics of κ_t (55), the dynamics of automation (63), and the resource constraint $(1-\kappa_t)Y_t = c_t L_t + D_t^v + D_t^h$. Thus, the social planner's solution can be derived by setting up the following current-value Hamiltonian:

$$\hat{H}_t = \ln c_t + \lambda_t \left[h^{\alpha-1}\xi^h(\chi_t^h)^\alpha - \phi\xi^v(\chi_t^v)^\alpha(\kappa_t)^{1-\alpha}\right] + \mu_t\left[-\xi^h h^\alpha(\chi_t^h)^\alpha\Gamma_t\right] + v_t\left[Y_t(1-\kappa_t-\chi_t^h-\chi_t^v) - c_t L\right], \tag{66}$$

where λ_t is the co-state variable associated with the constraint (55), μ_t is the co-state variable associated with the constraint (63), and v_t is the multiplier of resource constraint. The first-order conditions in this problem are given by

$$\frac{\partial \hat{H}_t}{\partial c_t} = 0 \Rightarrow v_t = \frac{1}{Lc_t}, \tag{67}$$

$$\frac{\partial \hat{H}_t}{\partial \chi_t^h} = 0 \Rightarrow \lambda_t \alpha h^{\alpha-1}\xi^h(\chi_t^h)^{\alpha-1} - \mu_t\alpha\xi^h h^\alpha(\chi_t^h)^{\alpha-1}\Gamma_t - v_t Y_t = 0, \tag{68}$$

$$\frac{\partial \hat{H}_t}{\partial \chi_t^v} = 0 \Rightarrow -\lambda_t\phi\alpha\xi^v(\chi_t^v)^{\alpha-1}\kappa_t^{1-\alpha} - v_t Y_t = 0, \tag{69}$$

$$\frac{\partial \hat{H}_t}{\partial \kappa_t} = \lambda_t\Big[-\phi\xi^v(\chi_t^v)^\alpha(1-\alpha)\kappa_t^{-\alpha}\Big] + v_t\Big[\frac{Y_t}{\sigma(1-\kappa_t)}(1-\kappa_t-\chi_t^h-\chi_t^v) - Y_t\Big] = \rho\lambda_t - \dot{\lambda}_t, \tag{70}$$

$$\frac{\partial \hat{H}_t}{\partial \Gamma_t} = \mu_t\big[-\xi^h h^\alpha(\chi_t^h)^\alpha\big] + v_t\frac{(1-\sigma)Y_t}{\sigma\Gamma_t}(1-\kappa_t-\chi_t^h-\chi_t^v) = \rho\mu_t - \dot{\mu}_t \tag{71}$$

with the transversality conditions $\lim_{t\to\infty} e^{-\rho t}\lambda_t\kappa_t = 0$, and $\lim_{t\to\infty} e^{-\rho t}\mu_t\Gamma_t = 0$. Multiplying (71) by Γ_t and multiplying (63) by μ_t, respectively, then combining (67), we can obtain a differential equation for the jump variable $\mu_t\Gamma_t$ such that $\dot{\mu}_t\Gamma_t + \mu_t\dot{\Gamma}_t = \rho\mu_t\Gamma_t - (1-\sigma)/\sigma$, which implies that $\mu_t\Gamma_t$ would jump to its steady-state value given by $(1-\sigma)/(\sigma\rho)$. Moreover, substituting (69) into (68) yields

$$\lambda_t\left[\alpha h^{\alpha-1}\xi^h(\chi_t^h)^{\alpha-1} + \phi\alpha\xi^v(\chi_t^v)^{\alpha-1}\kappa_t^{1-\alpha}\right] = \alpha\xi^h h^\alpha(\chi_t^h)^{\alpha-1}\left(\frac{1-\sigma}{\sigma\rho}\right). \tag{72}$$

Substituting (67) into (69) yields

$$\lambda_t \left[-\phi\xi^v(\chi_t^v)^\alpha(1-\alpha)\kappa_t^{-\alpha}\right] = \frac{1}{1-\chi_t^v-\chi_t^h-\kappa_t}, \tag{73}$$

and using (67) and (70), the dynamics of λ_t can be written as:

$$\dot{\lambda}_t = \lambda_t \left[\rho + \phi\xi^v(\chi_t^v)^\alpha(1-\alpha)\kappa_t^{-\alpha}\right] + \frac{1}{1-\chi_t^v-\chi_t^h-\kappa_t} - \frac{1}{\sigma(1-\kappa_t)}. \tag{74}$$

Therefore, the dynamical system of the social optimum is characterized by the dynamics of κ_t (55) and the dynamics of λ_t (74), together with two static equations (72) and (73). Based on this dynamical system, we can use relaxation algorithm to obtain the time paths of optimal allocation variables (i.e., capital-investment ratio $\chi_{sp,t}^I \equiv \kappa_{sp,t}$, automation investment ratio $\chi_{sp,t}^h$ and vertical innovation investment ratio $\chi_{sp,t}^v$).

7.4.2 First-best policy mix

Recall that the labor share of income S_L is 60%, which implies an initial value of $\kappa_0 = 37.18\%$. Given this initial condition, the decentralized equilibrium allocations of χ^I, χ^h, and χ^v are represented by the yellow dotted lines in Figure 10a, Figure 10b, and Figure 10c, respectively. These allocations remain constant over time because the economy is initialized in the decentralized steady state. Although the decentralized economy is already in its steady state, the socially optimal allocation is not. Rather, the socially optimal allocations are on their transition paths, as indicated by the solid blue lines in Figure 10. Specifically, Figure 10a illustrates that the socially optimal capital-investment ratio χ_{sp}^I starts below its long-run optimal level and gradually rises toward ${\chi_{sp}^I}^*$. In contrast, the socially optimal automation investment share χ_{sp}^h, shown in Figure 10b, initially exceeds its long-run optimum before declining toward ${\chi_{sp}^h}^*$, which remains consistently higher than its decentralized counterpart. Finally, as shown in Figure 10c, the optimal allocation χ_{sp}^v begins below its long-run optimum and steadily converges to the steady-state level ${\chi_{sp}^v}^*$.

A comparison of the equilibrium resource allocations with their first-best counterparts reveals that the inefficiencies in the decentralized equilibrium stem from three layers of distortions. The first pertains to the capital-investment ratio χ^I. Recall that this ratio is pinned down by the technology indices $\beta_t Z_t^{-1}$ (i.e., κ_{sp}) in the social optimum where no policy interventions are involved, while it is distorted to $[(1-s^x)\mu]^{\frac{1}{\sigma-1}}\kappa$ in decentralized equilibrium. To replicate the first-best outcome, the resource allocation on horizontal innovation and vertical innovation, χ^h and χ^v, must match their socially optimal levels, χ_{sp}^h and χ_{sp}^v simultaneously. Given the law of motion for κ in (55), this alignment requires $\kappa = \kappa_{sp}$. This implies that the distortion in the capital-investment ratio can be eliminated by setting a time-invariant intermediate-good subsidy such that $(1-s^x)\mu = 1$. Intuitively, an increase in patent breadth μ allows capital-good producers to charge higher prices, which in turn lowers the demand for capital goods and reduces the capital-investment ratio. An intermediate-good subsidy s^x offsets this markup distortion by stimulating

demand for capital goods and restoring the socially appropriate incentive to produce intermediate goods with capital. In this way, the efficiency loss resulting from resource misallocation in I_t/Y_t can be rectified.

The second distortion pertains to the allocation of the aggregate horizontal innovation expenditure share χ^h. Allocative inefficiencies emerge when the decentralized equilibrium allocates excessive or insufficient resources relative to the social optimum. The distortion stems from externalities associated with automation that are not internalized in individual firms. Equation (23) illustrates the interplay between two opposing effects. On the one hand, the duplication externality incentivizes overinvestment in automation, as firms redundantly develop competing technologies. On the other hand, the increasing complexity of the automation process raises the cost of horizontal innovation as automation advances, discouraging investment and resulting in underinvestment. Moreover, because firms fail to internalize the positive externality of automation on aggregate economic growth, horizontal innovation remains underinvested relative to the social optimum. In our calibration, the force of underinvestment predominates, as indicated by the wedge between the decentralized equilibrium allocation χ^h and the socially optimal allocation χ^h_{sp} in Figure 10b. Accordingly, to neutralize this distortion and align the decentralized equilibrium with the social optimum, dynamic automation subsidies are required to ensure that $\chi^h = \chi^h_{sp}$.

The third distortion pertains to the share of aggregate vertical innovation expenditures relative to total output, χ^v. As in standard Schumpeterian growth models, private R&D investment is subject to multiple externalities, including intertemporal knowledge spillovers, business-stealing effects, and surplus appropriability problems. These externalities give rise to a wedge between the decentralized equilibrium allocation χ^v and the socially optimal allocation χ^v_{sp}, as illustrated in Figure 10c. This wedge between χ^v_{sp} and χ^v indicates a systemic underinvestment in vertical innovation. To remedy these R&D distortions, dynamically R&D subsidies are required to ensure that $\chi^v = \chi^v_{sp}$.

It is worth pointing out that although these distortions can be conceptually distinguished, the distortions associated with automation and vertical innovation cannot be cleanly separated along the transition path. Horizontal innovation and vertical innovation interact along both technological and resource dimensions. First, automation expands the set of industries eligible for vertical innovation, thereby affecting the investment in vertical innovation. Conversely, the productivity gains generated by vertical innovation shift the incentives for firms to invest in further automation. Moreover, these two types of innovation compete for a common pool of resources. Given these technological interdependencies and the resulting crowding-out effects, policies targeting a single distortion may generate significant spillovers on the other. Consequently, automation and R&D subsidies must be coordinated to jointly address both distortions and achieve the social optimum.

By replicating the trajectories of optimal allocations χ^I_{sp}, χ^h_{sp} and χ^v_{sp}, we can numerically trace the evolution of the optimal policy over time for $\kappa_0 = 0.3718$, as depicted in Figure 11. First, Figure 11a indicates that the optimal intermediate-good subsidy s^x_{sp} remains constant over time at approximately 16.6%, which coincides with the welfare-maximizing rate identified in both the

steady-state and transitional welfare analyses. This result suggests that the distortion associated with the capital-investment ratio is static.

Figure 11b and Figure 11c show that the optimal automation subsidy s^h_{sp} and R&D subsidy s^v_{sp} are both initially high and then decline over time toward their respective long-run optimal levels. These declining trajectories are driven by the dynamic evolution of the wedges between the socially optimal and decentralized allocations, as well as the strategic complementarity between the two forms of innovation. Initially, the social planner implements a substantial automation subsidy to incentivize investment in automation. Simultaneously, a relatively high R&D subsidy is deployed to offset the crowding-out effect of automation expansion and to foster vertical innovation in its early stage. As the automation investment share gradually approaches its long-run optimal level, the marginal benefit of automation subsidies diminishes. At the same time, the expanded industrial base generates stronger endogenous incentives for vertical innovation, narrowing the wedges between the decentralized and optimal allocations for both types of innovation. As a result, both s^h_{sp} and s^v_{sp} decrease over time.

Using the dynamic optimal policies derived from the preceding analysis, we simulate the equilibrium trajectory under this policy regime. The resulting equilibrium allocations, represented by the orange dashed lines in Figure 10, perfectly replicate the first-best paths, demonstrating that the coordinated policy mix successfully eliminates the distortions.

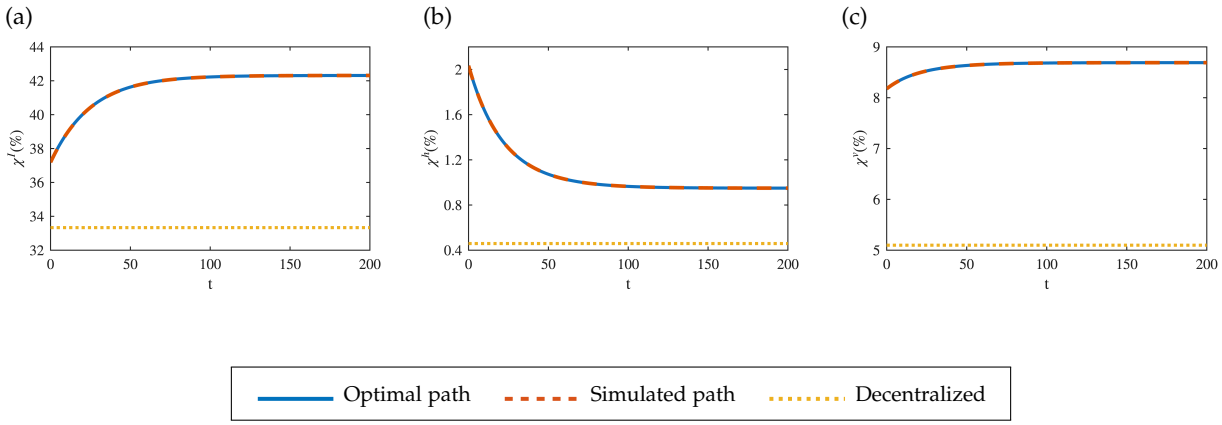


Fig. 10. Optimal, simulated, and decentralized paths of κ_{sp}, χ^h_{sp}, χ^v_{sp} ($\kappa_0 = 0.3718$).

7.4.3 Lower labor share of income

Motivated by the well-documented decline in the labor share of income (see Grossman and Oberfield 2022 and Bergholt *et al.* 2022), we now conduct a counterfactual exercise in which the initial value of κ_0 is set to 0.4648, where the economy is located in a lower labor share $S_L = 50\%$. This exercise allows us to examine how the first-best allocation and the associated optimal policy would differ under the alternative initial condition. Figure 12 reports the transition paths of the first-best allocation and the decentralized equilibrium, while Figure 13 presents the trajectories of corresponding optimal policy instruments.

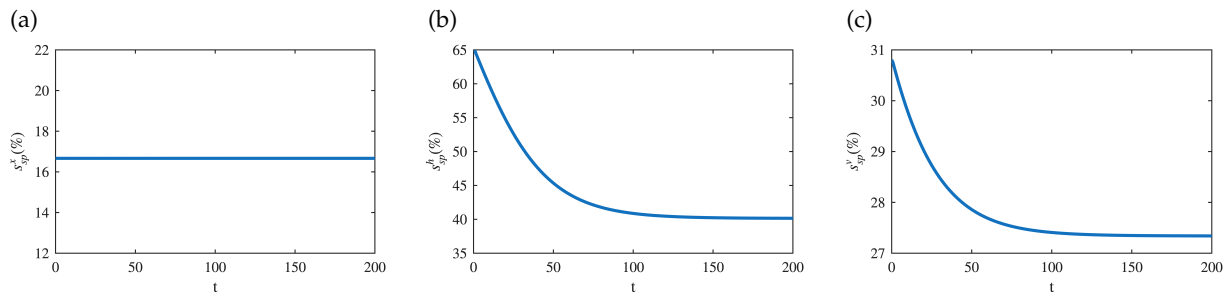


Fig. 11. Optimal path for s^x, s^h, s^v ($\kappa_0 = 0.3718$).

As shown in Figure 12a, the decentralized level of capital-investment ratio χ^I and its optimal counterpart χ^I_{sp} both gradually decrease towards their respective steady-state level. Nevertheless, the equilibrium allocation χ^I is always below the first-best level χ^I_{sp} during the entire time path. This is caused by the distortion arising from the monopoly pricing in capital-based intermediate good sectors. To remove this static distortion, the social planner adopts a constant intermediate-good subsidy, as shown in Figure 13a.

Interestingly, Figure 12b illustrates that the equilibrium automation investment ratio χ^h is initially above its socially optimal counterpart χ^h_{sp}, which suggests that the model economy over-allocates resources to automation when the economy features a lower labor share. To correct the excessive automation, the social planner initially imposes an automation tax of -33.67%, as indicated in Figure 13b. Over time, however, the wedge between the decentralized and socially optimal allocations gradually shrinks, so the required automation tax becomes smaller in magnitude. Once the wedge reverses, that is, the socially optimal automation investment ratio exceeds its equilibrium counterpart, the planner turns to subsidize automation. As this positive wedge continues to widen during the transition path, the optimal automation subsidy increases correspondingly, until the economy converges to its long-run optimum.

As for the dynamics of allocations on vertical innovation, Figure 12c shows that the equilibrium allocation χ^v and the socially optimal allocation χ^v_{sp} both gradually decline to their long-run level. More importantly, the first-best allocation stays above the equilibrium allocation throughout the transition path, implying a persistent underinvestment in vertical innovation in the economy. Therefore, the social planner subsidizes vertical innovation along the entire transition path. As shown in Figure 13c, the optimal R&D subsidy gradually rises before converging to its steady-state level, so that the increasing gap between χ^v_{sp} and χ^v can be eliminated.

In summary, this counterfactual exercise reveals that while the intermediate-good subsidy remains stable, the optimal policies for automation and vertical innovation exhibit distinct trajectories. When the economy is characterized by a lower initial labor share of income, the optimal automation policy is non-monotonic: it begins as a substantial tax to curb excessive automation before transitioning into a subsidy that increases toward its long-run optimal level. In contrast, the subsidy for vertical innovation increases monotonically. These findings underscore that the design of the optimal policy regime is highly sensitive to the economy's initial conditions.

While we have characterized the dynamically optimal policy mix, it may be practically chal-

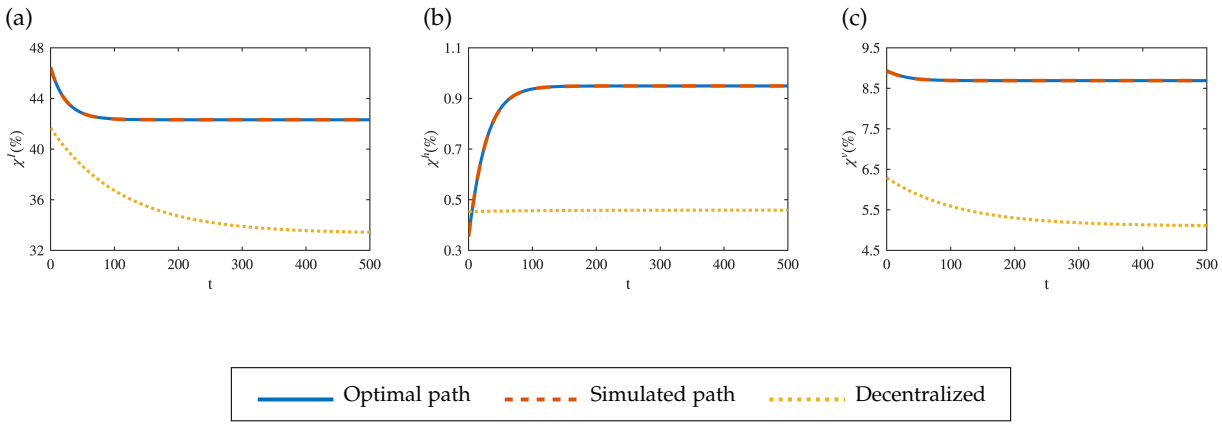


Fig. 12. Optimal, simulated, and decentralized paths of κ_{sp}, χ^h_{sp}, χ^v_{sp} ($\kappa_0 = 0.4648$).

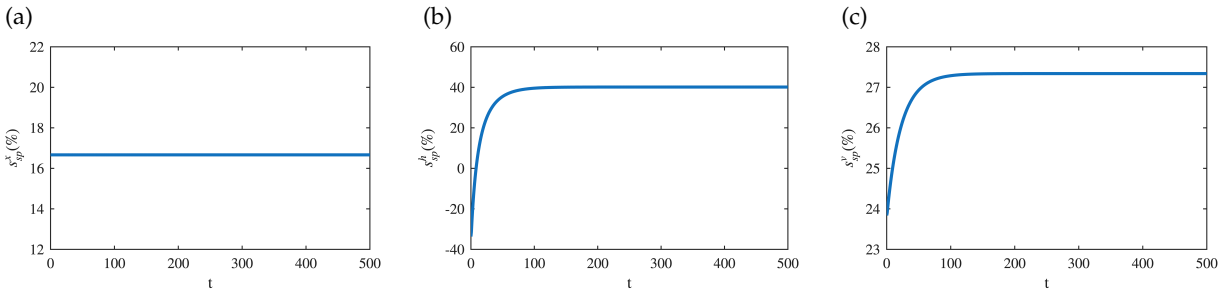


Fig. 13. Optimal path for s^x, s^h, s^v ($\kappa_0 = 0.4648$).

lenging for policymakers to identify the economy's exact position along the transition path. Consequently, a simpler alternative would be the adoption of a time-invariant policy constrained to the long-run optimal levels. To evaluate this alternative, the following subsection compares the trajectories of the relevant variables under these two regimes and quantifies the welfare loss incurred by implementing the constrained policy relative to the dynamically first-best benchmark.

7.4.4 Constrained optimal policy versus first-best policy

In this subsection, we quantitatively examine the evolution of key allocations under two distinct policy regimes: the constrained optimal policy, in which the long-run optimal rates are implemented from the outset, and the first-best policy, which follows the dynamically optimal subsidy path. Comparing these trajectories provides insight into the distortions introduced by the constrained regime relative to the first-best benchmark. Subsequently, we evaluate the feasibility of the constrained policy by quantifying the intertemporal welfare loss associated with it. Consistent with the previous analysis, we consider two scenarios: first, where the initial value κ_0 is below the optimal steady-state level (corresponding to the calibrated moment), and second, where κ_0 is set above the optimal steady-state level, resulting in a lower initial labor share of income.

We now explore the transition paths of relevant variables for the initial condition at $\kappa_0 = 0.3718$, as illustrated in Figure 14. Figure 14a shows that while capital-investment ratio χ^I starts at the same initial value and converges to the same steady-state level under both policy regimes, it follows a more gradual trajectory under the constrained optimal policy, resulting in slower convergence to the steady state.

Recall that the dynamically first-best automation subsidy initially exceeds the long-run optimum and then declines over time (See Figure 11b). Holding the automation subsidy at the long-run optimal level therefore yields insufficient incentives for automation, producing an underallocation of resources to automation relative to the first-best path in the early stages of the transition, as illustrated in Figure 14b. As the dynamically optimal subsidy approaches the long-run level imposed under the constrained policy, the allocation gap gradually narrows and eventually disappears. Analogously, the dynamically first-best R&D subsidy falls toward its long-run optimal level during the transition (See Figure 11c). Imposing a time-invariant R&D subsidy equal to the long-run optimal level therefore generates a lower initial investment in vertical innovation, but the deviation from the first-best trajectory diminishes over time (See Figure 14c).

Finally, Figure 14d contrasts the (log) levels of consumption under the two policy regimes. Under the constrained optimal policy, consumption is initially higher because a larger share of resources is allocated to current consumption rather than to automation and vertical innovation. This lower investment in innovation depresses the growth rate of consumption relative to the first-best regime, causing the initially higher consumption path to be eventually overtaken by the first-best trajectory as the transition unfolds. The constrained policy therefore entails an intertemporal tradeoff: it raises current consumption by diverting resources away from innovation, but at the cost of weaker long-run economic growth. Consequently, implementing the constrained policy instead of the first-best benchmark leads to a welfare loss equivalent to a permanent re-

duction in per capita consumption of 0.35%; this non-negligible loss underscores the superiority of the dynamically optimal policy regime.

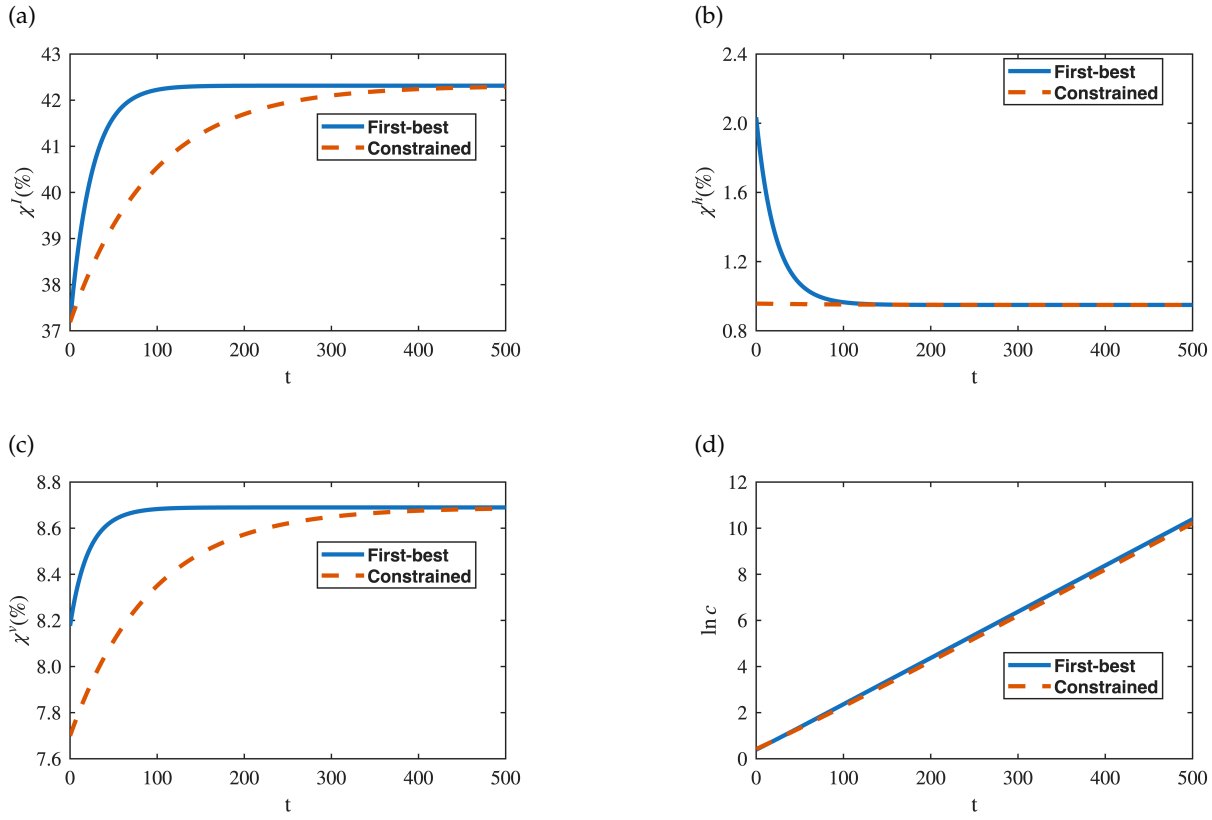


Fig. 14. Constrained optimal policy versus first-best policy ($\kappa_0 = 0.3718$)

Next, we consider the alternative case where the initial value of κ is set to 0.4648, with the corresponding trajectories of key variables displayed in Figure 15. In this scenario, the implementation of the long-run optimal policy fails to replicate the first-best transition dynamics; instead, it induces persistent misallocations in capital investment, automation, and vertical innovation throughout the transition path.

Specifically, under the constrained optimal policy, the capital-investment ratio decreases toward the steady state along a flatter trajectory, as depicted by the dashed orange line in Figure 15a. While the long-run optimal intermediate-good subsidy rate aligns with its first-best counterpart, which thereby removes the distortion stemming from monopoly power, the externalities associated with automation and R&D externalities persist. Consequently, the capital-investment ratio χ^I stays above its socially optimal level χ^I_{sp} throughout most of the transition, until they eventually converge to the common steady-state value.

Given the initial condition $\kappa_0 = 0.4648$, the economy starts with overinvestment in automation. In this case, applying the long-run optimal automation subsidy throughout the entire transition fails to correct the initial distortion; instead, it further diverts resources toward au-

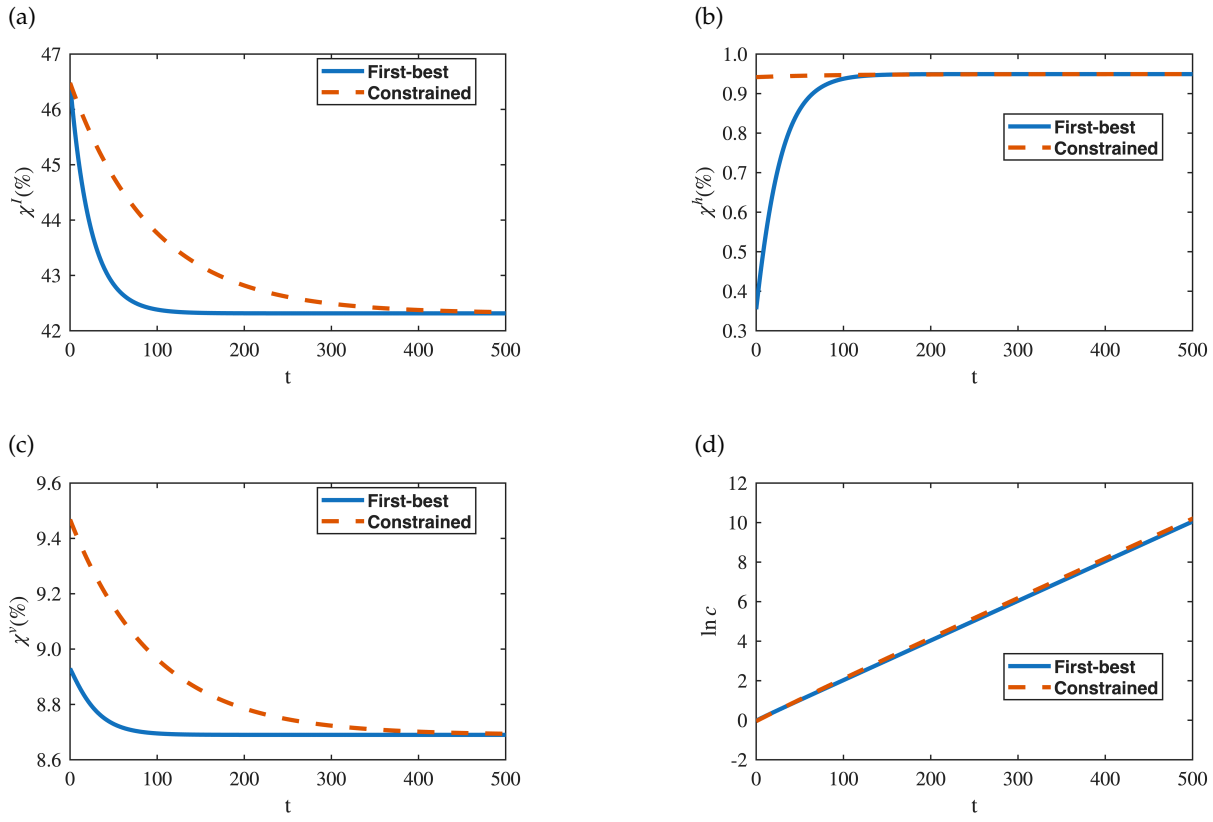


Fig. 15. Constrained optimal policy versus first-best policy ($\kappa_0 = 0.4648$)

tomation, as illustrated in Figure 15b. Likewise, the long-run optimal R&D subsidy over-corrects the distortion in vertical innovation, generating overinvestment in χ^v (See Figure 15c). These misallocations gradually vanish as the dynamically optimal subsidies converge to their long-run levels, so that the constrained and first-best allocations eventually coincide at the same steady state level.

The aforementioned misallocations in χ^I, χ^h, and χ^v have significant implications for consumption dynamics, as depicted in Figure 15d. Under the constrained optimal policy, excessive investment in capital production, automation, and vertical innovation crowds out contemporaneous consumption. While the increased investment in innovations increases the rate of consumption growth in the later stage of the transition, the gains are insufficient to offset the initial consumption loss. Quantitatively, relative to the first-best policy, implementing the constrained optimal policy results in a welfare loss equivalent to a permanent reduction in per capita consumption of 0.27%. Although this welfare loss is slightly smaller than the 0.35% welfare loss in the previous case, it remains far from trivial, highlighting that constrained long-run optimal policies should be adopted with caution.

In summary, regardless of initial conditions, the constrained optimal policy fails to eliminate distortions and replicate the first-best allocations. The welfare loss incurred by implementing a constrained steady-state policy rather than the dynamically first-best optimal policy is economically non-negligible. Therefore, while the constrained long-run optimal policy may be attractive for its simplicity, it is fundamentally suboptimal and may lead to significant economic inefficiencies.

8 Conclusion

This study investigates the growth and welfare implications of a subsidization policy regime within a growth framework characterized by capital-embodied technical change. In our model, capital inputs advance along two technological margins: automation and productivity improvement. The policy regime consists of three instruments: subsidies for the purchase of intermediate goods, subsidies for automation, and subsidies for vertical innovation.

We first theoretically analyze the steady-state effects of these policies on economic growth and the labor share of income. We find that intermediate-good and automation subsidies both promote long-run growth, whereas R&D subsidies hinder it. This result stems from the fact that balanced growth in the model is primarily driven by automation; thus, policies that stimulate automation accelerate long-run growth, while R&D subsidies, despite encouraging vertical innovation, crowd out automation and consequently depress growth. Regarding the labor share of income, intermediate-good and R&D subsidies have a positive effect, owing to a less-than-unitary elasticity of substitution between labor and capital. Conversely, the automation subsidy reduces the labor share, as it facilitates the replacement of labor by capital.

Calibrating the model to US data reveals that transition dynamics are pivotal for policy evaluation. While the intermediate-good subsidy exhibits no transitional volatility, the R&D subsidy produces a striking temporal trade-off: it stimulates short-term growth despite hindering

long-term expansion. This divergence fundamentally alters welfare prescriptions. For instance, automation subsidies, which appear suboptimal in the steady state, are supported by transitional welfare analysis due to significant short-term gains. Similarly, the optimal R&D subsidy is more modest in the transition than in the steady state, reflecting the intertemporal trade-off between immediate losses and future gains.

Finally, we explore the optimal policy mix required to replicate the first-best outcome. We demonstrate that correcting distortions stemming from monopoly power and the externalities associated with automation and vertical innovation requires time-varying, first-best policies. Our quantitative analysis characterizes the transition paths of these instruments and shows that replacing them with simplified, constrained long-run optimal policies entails non-negligible welfare losses.

This study provides a comprehensive analysis of how distinct subsidy policies impact growth, labor share, and welfare in a capital-embodied technical change framework. To isolate the effects of the policy instruments, we assume that subsidies are financed through non-distortionary lump-sum taxes. While this simplification is standard in the literature, it abstracts from the fact that government revenue is typically collected through distortionary taxes that also influence resource allocation. Therefore, incorporating distortionary taxation, such as capital or labor income taxes, represents a promising direction for future research to better understand the interaction between subsidy and fiscal policy.

A Appendix

A.1 Proof of BGP1

Differentiating the first BGP condition (i.e., $\beta_t Z_t^{-1} = \kappa$) and using Leibniz's rule yields

$$\int_0^{\beta_t} \frac{\dot{z}_t(i)}{z_t(i)^2} di = \frac{\dot{\beta}_t}{z_t(\beta_t)}. \tag{A.1}$$

For the left-hand side of equation (A.1), recall that the vertical innovation sector has a step size of ϕ with a hazard rate q^v. Given $z_t(i)$, we have $\dot{z}_t(i)/z_t(i) = \phi q^v$ in expectation. Thus, we can express the integral as follows:

$$\int_0^{\beta_t} \frac{\dot{z}_t(i)}{z_t(i)^2} di = \phi q^v \int_0^{\beta_t} \frac{1}{z_t(i)} di = \kappa \phi q^v. \tag{A.2}$$

The second condition for the balanced growth path (BGP) states that $g_{1-\beta_t} = -q^h$. It follows that

$$g_{\beta_t} = \frac{\dot{\beta}_t}{\beta_t} = \frac{1-\beta_t}{\beta_t} \frac{\dot{\beta}_t}{1-\beta_t} = \frac{1-\beta_t}{\beta_t} q^h. \tag{A.3}$$

This implies that automation occurs at the rate $\dot{\beta}_t = (1-\beta_t) q^h$ on the BGP. Using (24) , the right-hand side of (A.1) can be written as q^h/h. Consequently, we arrive at:

$$\kappa \phi q^v = \frac{q^h}{h}. \tag{A.4}$$

Referring back to Lamma 1, we can reexpress the first BGP condition as:

$$\beta_t Z_t^{-1} = \frac{1}{h\phi} \left(\frac{1-s^h}{1-s^v} \right)^{\frac{\alpha}{\alpha-1}} \left(\frac{\xi^h}{\xi^v} \right)^{\frac{1}{1-\alpha}} \tag{A.5}$$

A.2 Proof of Lemma 2

Adoption of an automated technology requires that its unit cost, $\psi / z_t(i)^{\frac{\sigma-1}{\sigma}}$, be lower than unit cost of labor, w_t/A, as imposed by the condition (8). The adoption condition is automatically satisfied for every automated sectors once it holds for the automated sector with the lowest initial productivity. Notably, because initial automation productivity is declining over time, and because productivity can only increase after a technology has been automated, the sector with the lowest productivity level will be the newly automated one. Using the initial productivity for $z_t(\beta_t)$, (24) and the equilibrium wage expressed through the labor-share condition, (33), the technology-adoption condition can be written as

$$\frac{1}{1-s^x} \left[1 - [(1-s^x)\mu]^{\frac{\sigma}{\sigma-1}} \beta_t Z_t^{-1} \right] \frac{Y_t}{L_t} \geq \psi A h^{\frac{1-\sigma}{\sigma}} (1-\beta_t)^{\frac{1-\sigma}{\sigma}}. \tag{A.6}$$

Substituting (29) into this inequality yields

$$\frac{1}{1-s^x}\left[1-\left[(1-s^x)\mu\right]^{\frac{\sigma}{\sigma-1}}\beta_t Z_t^{-1}\right]^{\frac{\sigma-1}{\sigma}} \geq h^{\frac{1-\sigma}{\sigma}} \tag{A.7}$$

Finally, using the first condition for BGP, (40), we can obtain

$$h \geq (1-s^x)^{\frac{\sigma}{\sigma-1}}\left[1+\mu^{\frac{\sigma}{\sigma-1}}\frac{1}{\phi}\left(\frac{1-s^h}{1-s^v}\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}\right]. \tag{A.8}$$

Therefore, this condition ensures all automated technologies to be sufficiently productive to be adopted on the BGP, as stated in Lemma 2.

A.3 Sensitivity analysis

This appendix performs a sensitivity analysis by considering an alternative value of the elasticity of substitution between capital and labor at 0.5. Then we recalibrate the model to aggregate data of the US economy to redo a numerical analysis. The recalibrated parameter values are summarized in Table 3.

Table 3: Recalibration ($EoS = 0.5$)

ρ	v	ψ	μ	α	ϕ	s^x	s^v	s^h	σ	ξ^v	ξ^h	h
0.04	1	1	1.2	0.04	0.09	0	0.066	0.066	-1	0.220	0.046	2.282

Figure 16 simulates the long-run policy implications of each single policy instrument under the new value (i.e., 0.5) of the elasticity of substitution between labor and capital. It shows that the effects of individual policy on economic growth, the labor share of income and the steady-state welfare are consistent with the benchmark case, indicating that the long-run policy implications are robust to the alternative value of the elasticity of substitution.

Figure 17 simulates again the transitional dynamic effects of distinct policies on economic growth and the labor share of income, as well as the transitional welfare, under the new value of the elasticity of substitution. It is shown that the short-run policy implications align with the benchmark case. Specifically, the short-run effects of each single policy instrument are consistent with their long-run counterparts, except for the short-run growth effect of R&D subsidy. Moreover, when the value of elasticity of substitution between labor and capital decreases from 0.6 to 0.5, although the transitional welfare of s^x remains fully captured by the steady-state welfare (as shown in Figure 17c), the transitional welfare implication of automation subsidy and R&D subsidy becomes slightly different. On the one hand, as depicted in Figure 17f, the transitional welfare of s^h now features a higher optimal automation subsidy rate. On the other hand, Figure 17i shows that increasing subsidization on vertical innovation would incur welfare loss, resulting in a lower optimal R&D subsidy rate than that in the benchmark case, which is approximately at -16%. This suggests a moderate tax rate on vertical innovation.

Figure 18 simulates the trajectories of optimal policy mix over time under the alternative value of the elasticity of substitution. It is shown that the evolution of optimal policies follows the same pattern as in the benchmark case: the optimal intermediate-good subsidy s^x_{sp} remains time-invariant at 16.6, and the optimal automation subsidy s^h_{sp} and the optimal R&D subsidy s^v_{sp} both monotonically decline to their long-run optimal levels, respectively. However, in this case, the magnitude of s^h_{sp} becomes larger, whereas the magnitude of s^v_{sp} becomes smaller. These findings imply that the distortion arising from monopoly price is independent of the elasticity of substitution. In contrast, the distortion associated with automation appears to increase, while the distortion associated with R&D appears to decrease when the elasticity of substitution falls.

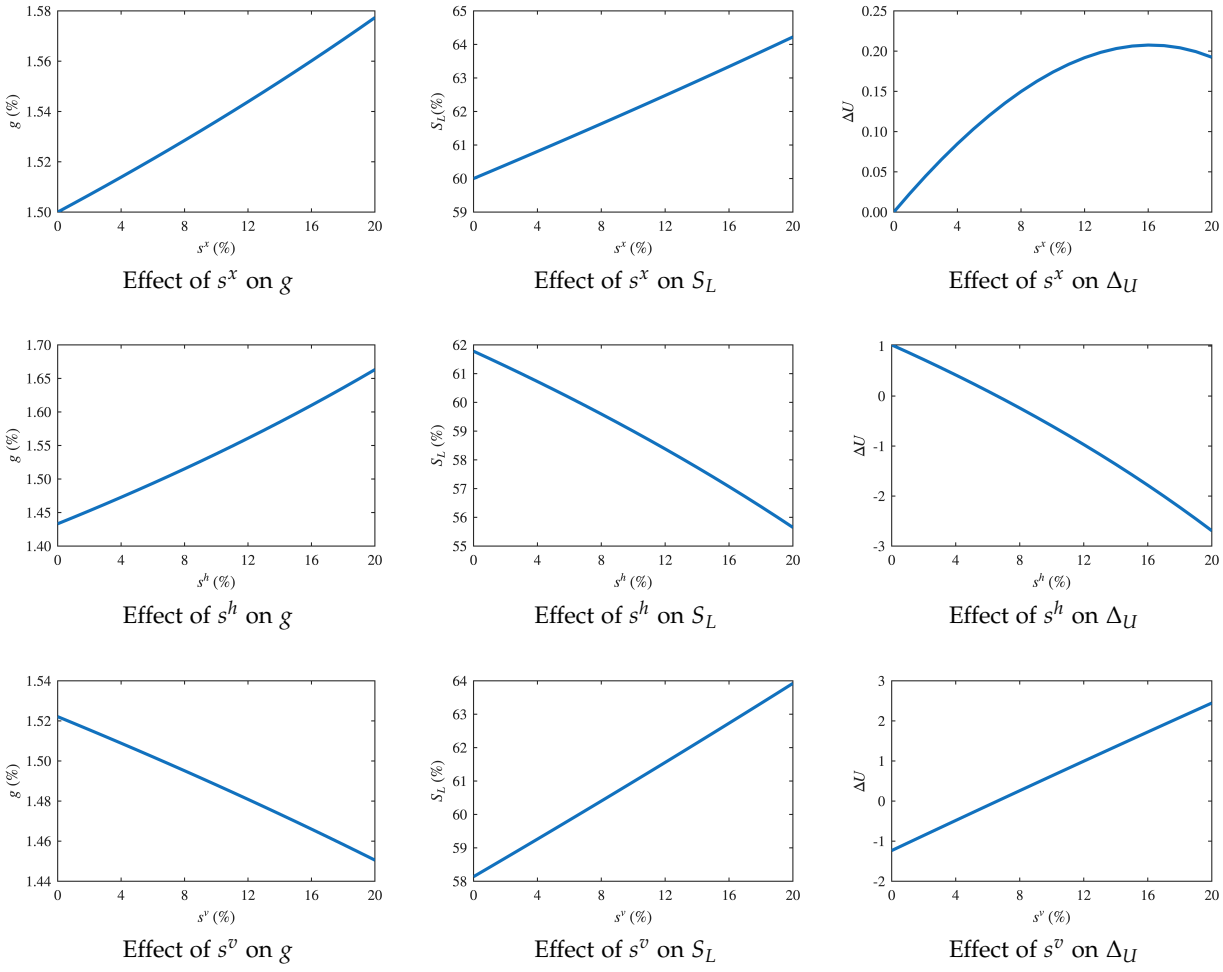
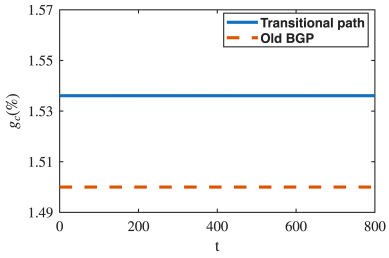
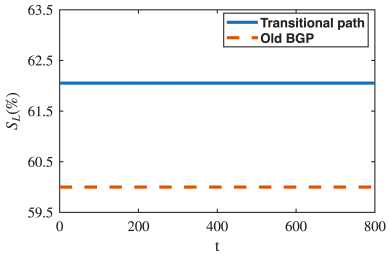


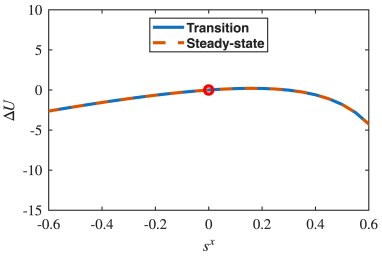
Fig. 16. Long-run policy implications of s^x, s^h, s^v ($EoS = 0.5$).



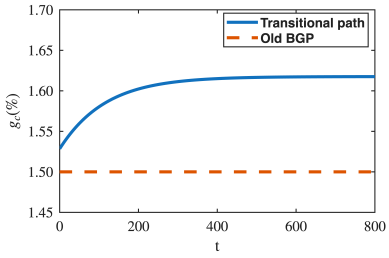
Dynamic effect of s^x on g_c



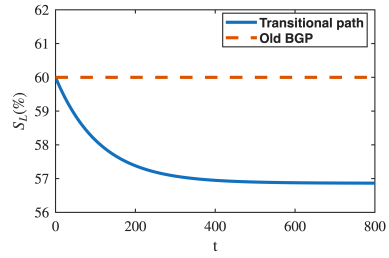
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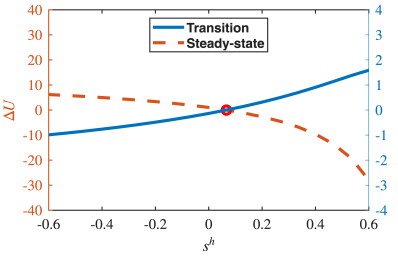
Transitional welfare of s^x



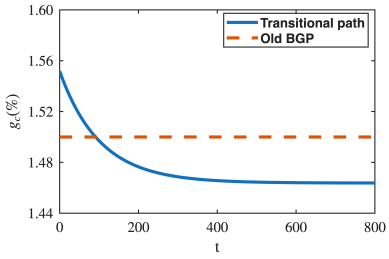
Dynamic effect of s^h on g_c



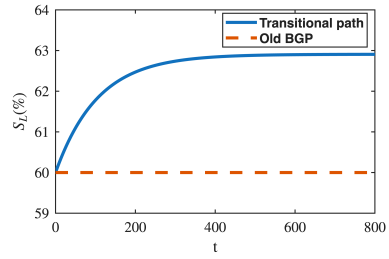
Dynamic effect of s^h on S_L



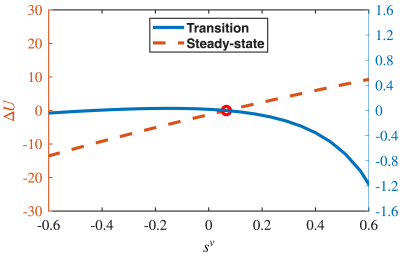
Transitional welfare of s^h



Dynamic effect of s^v on g_c

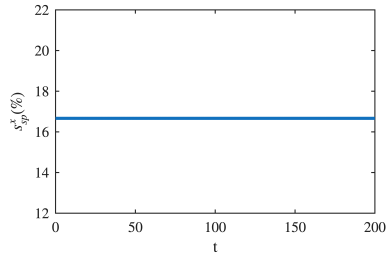


Dynamic effect of s^v on on S_L

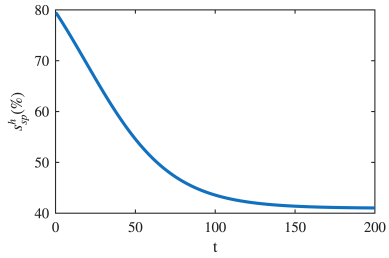


Transitional welfare of s^v

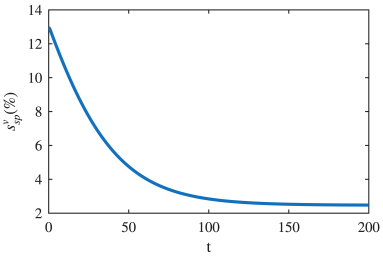
Fig. 17. Transitional dynamic effects of s^x, s^h, s^v ($EoS = 0.5$).



Optimal path for s^x



Optimal path for s^h



Optimal path for s^v

Fig. 18. Optimal path for s^x, s^h, s^v ($EoS = 0.5$).

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